Numerik für Simulation und Optimierung - Einführung

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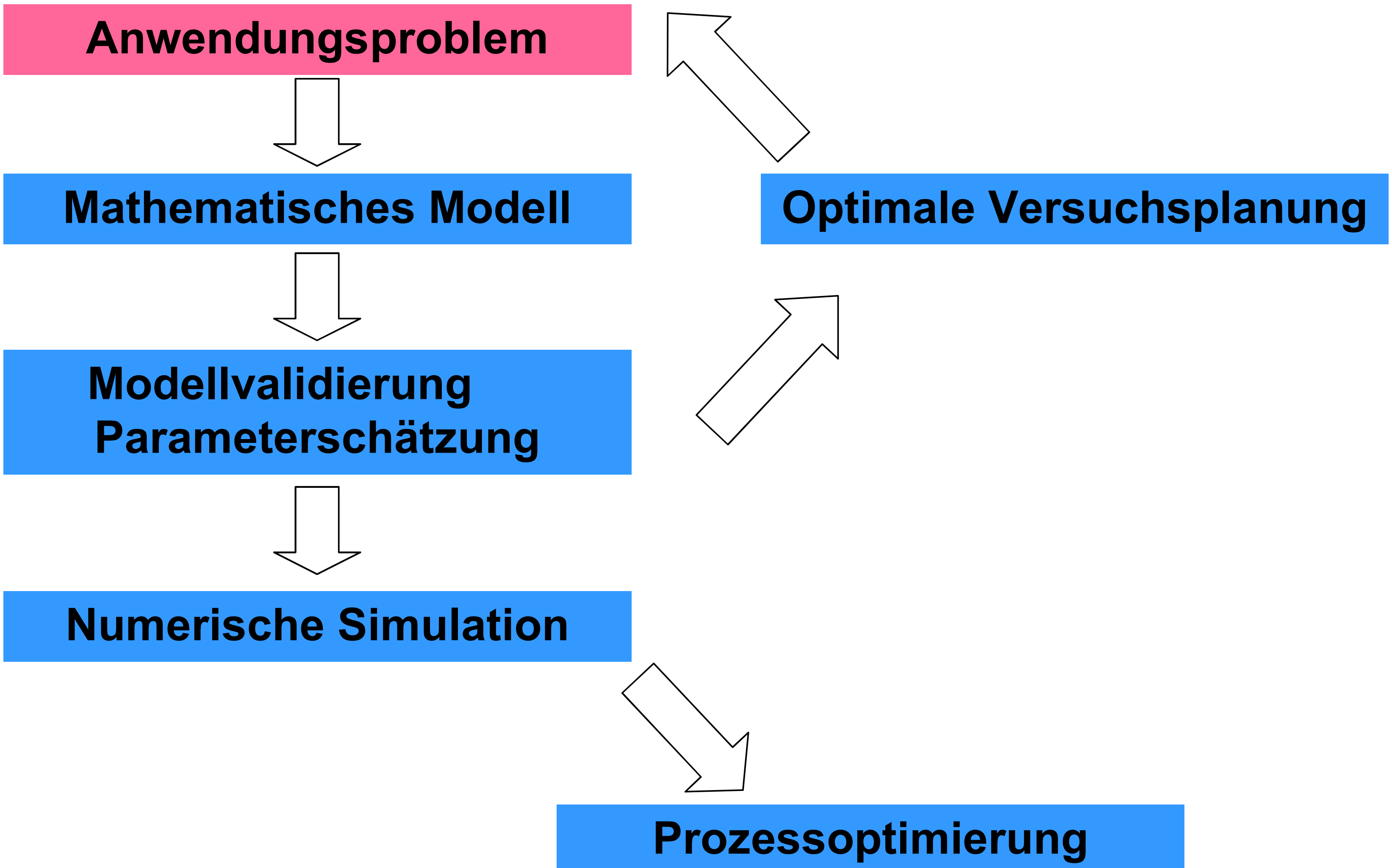
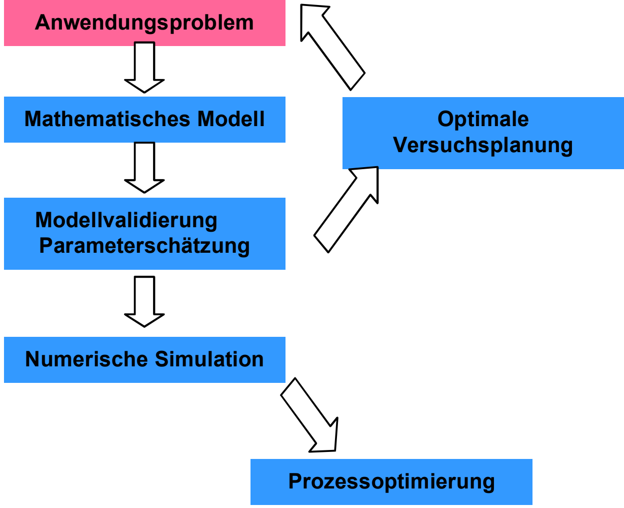
FB IV – Abt. Mathematik

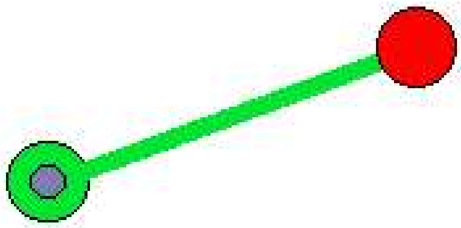
Universität Trier

Numerische Modellierung



Anwendungsproblem





Elastisches Pendel

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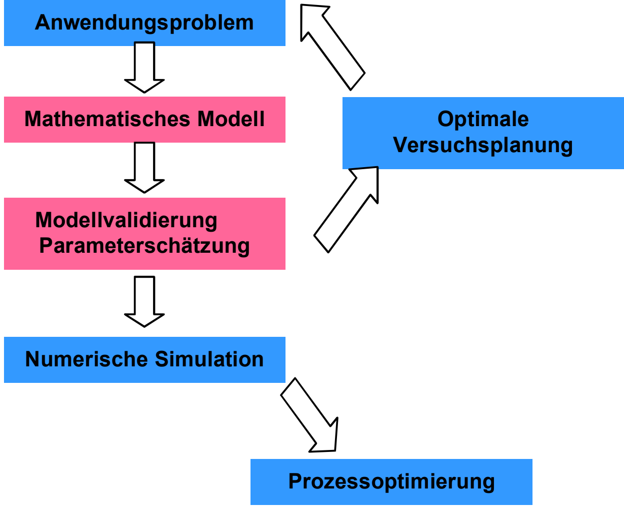
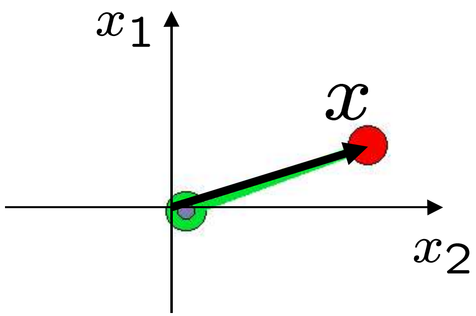
1. Mathematisches Modell

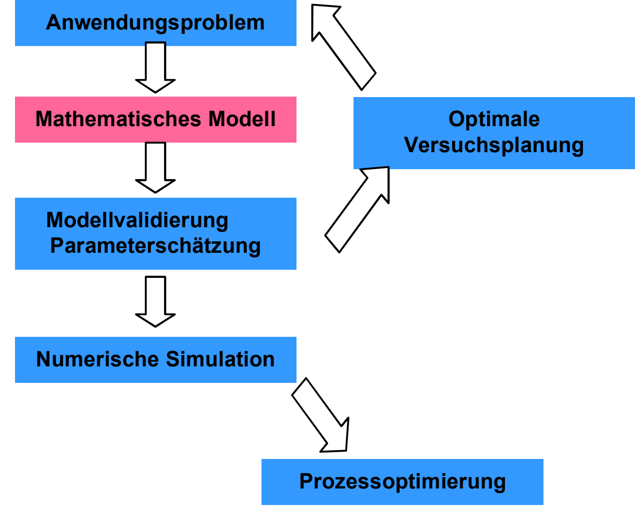


$$\begin{aligned} r_0 &= 0.23\text{m} \\ g &= 9.81\frac{\text{m}}{\text{s}^2} \\ m &= 80\text{g} \\ \gamma &= \frac{|x_2^{\text{rest}}| - r_0}{m\cdot g} \end{aligned}$$



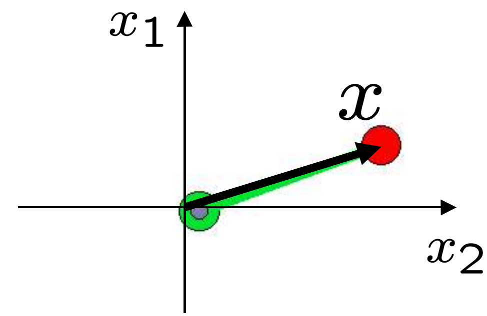
$$m\cdot\ddot{x} = \begin{pmatrix} 0 \\ -m\cdot g \end{pmatrix} - \gamma\cdot\frac{x}{\|x\|}\cdot(\|x\| - r_0)_+$$

Gravitation **Elastizität**



Modellannahmen

- **2D-Problem**
- **Ball = Massenpunkt**
- **Elastisches Band = Feder mit Konstante γ**
- **Kräfte:**
  - **Gravitation**
  - **Federspannung**



reicht das ?

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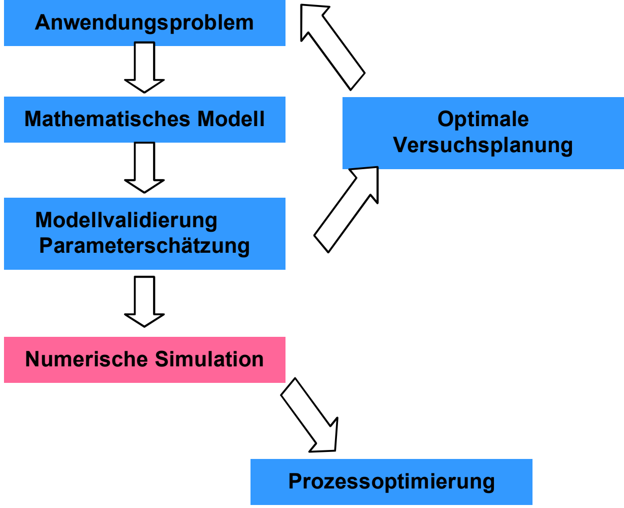
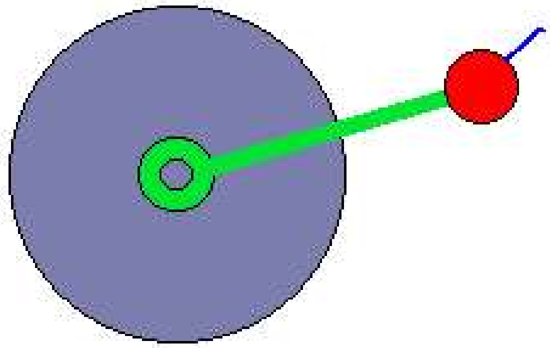
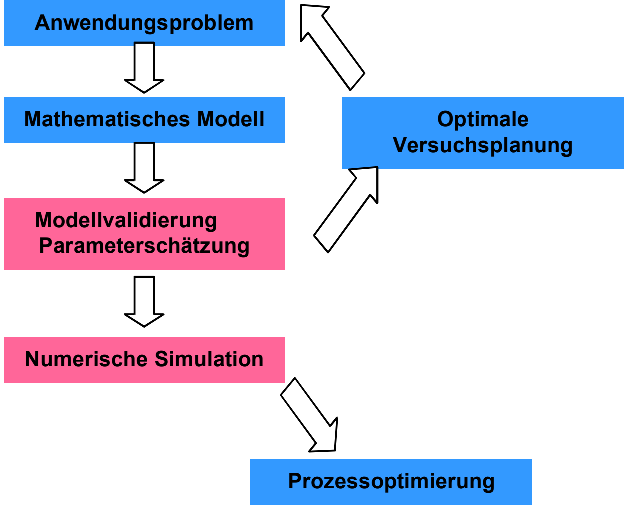


1. Simulation

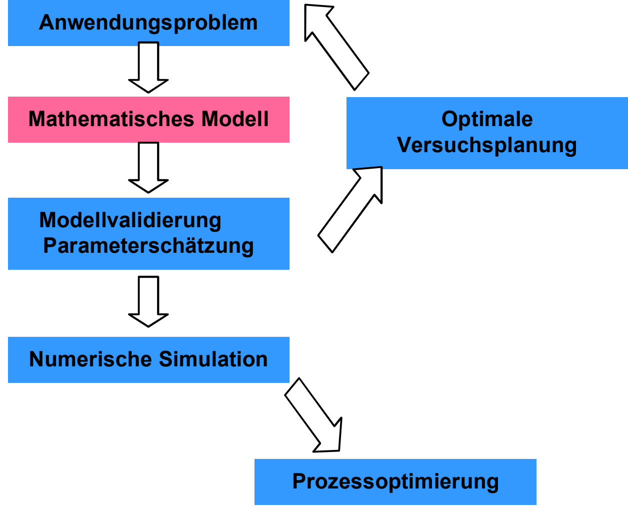


- *„Lissajous-Figur“*
- widerspricht Anschauung



Fehlerquellen

- Modell(ierungs-)fehler
- Daten-/Mess-fehler
- Diskretisierungsfehler:
  - Integrationsroutine für Anfangswertaufgabe
  - Schaltpunktbehandlung
  - (PDE-Diskretisierung)



2. Mathematisches Modell

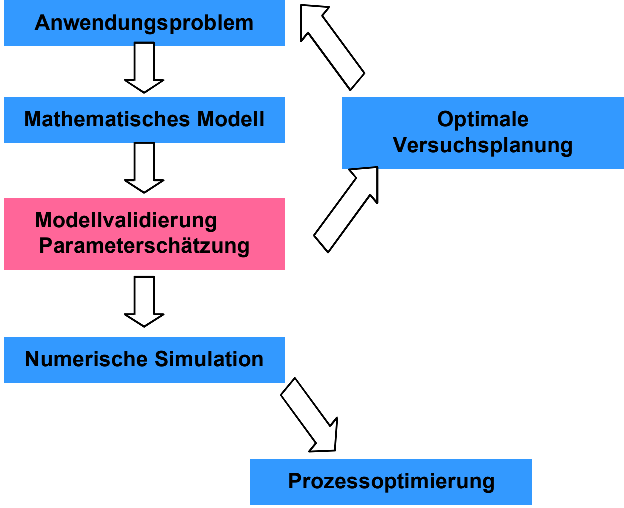
Luftwiderstand (linear, weil kleine Geschw.)

$$R_l = -c \cdot \dot{x}$$

innere Reibung des Bandes

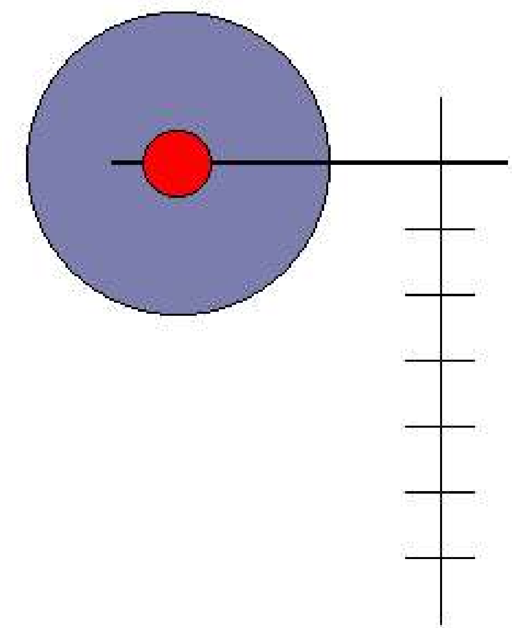
$$R_e = -\nu \cdot \left(\dot{x}, \frac{x}{\|x\|} \right) \frac{x}{\|x\|}$$

$$m \cdot \ddot{x} = \begin{pmatrix} 0 \\ -m \cdot g \end{pmatrix} - \gamma \cdot \frac{x}{\|x\|} \cdot (\|x\| - r_0)_+ + R_e + R_l$$



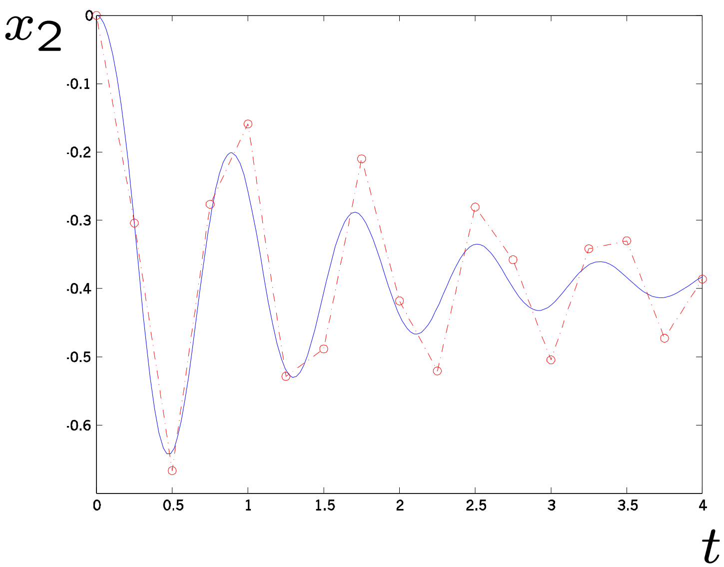
Parameterschätzung

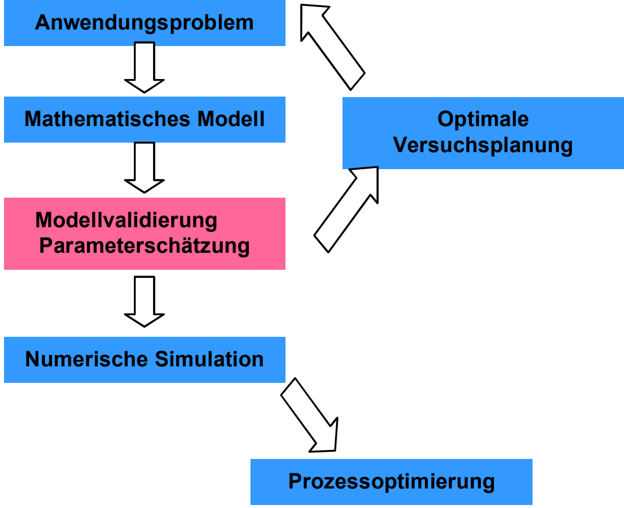
Parameter ν und c sind als effektive Größen nur am Gesamtobjekt zu untersuchen



Experiment

Parameterschätzung





Output least squares Ansatz

$$\min_{c,\nu} \sum_{i=1}^{n} \sigma_i^{-2} \cdot \left(x_2(t_i) - \hat{x}_{2,i}\right)^2$$

$$\text{unter} \quad \ddot{x} = f(x, c, \nu), \qquad x(0) = 0, \quad \dot{x}(0) = 0$$

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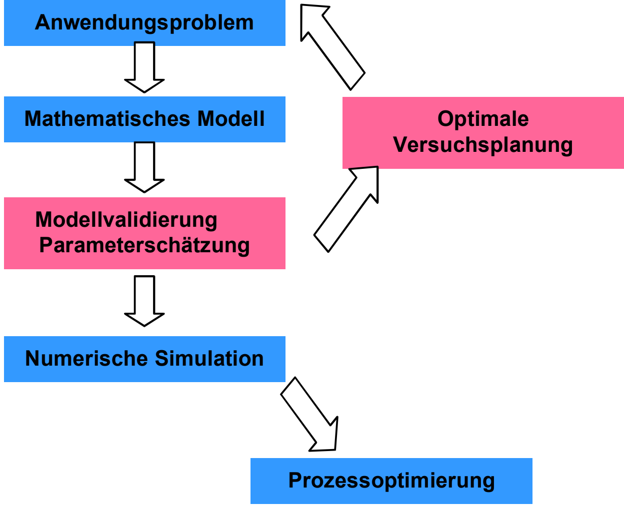


Sensitivitätsanalyse/ statistische Auswertung

Problem: **c und ν beeinflussen die Messdaten auf die gleiche Weise**

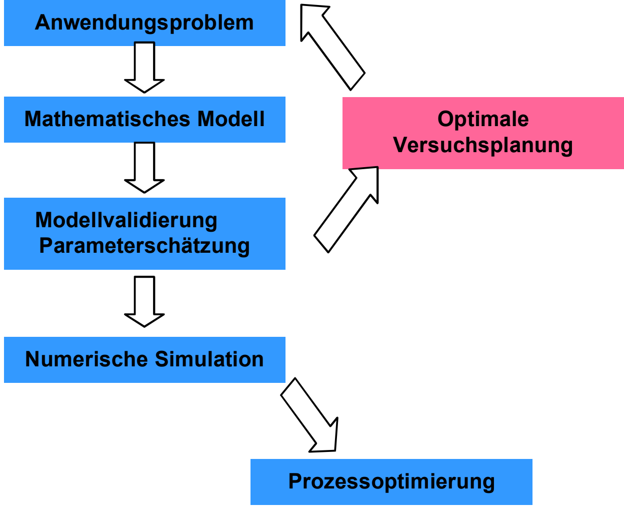
Die Matrix $$J := \frac{\partial}{\partial(c,\nu)} \begin{pmatrix} x_2(t_1, c, \nu) \\ \vdots \\ x_2(t_n, c, \nu) \end{pmatrix}$$

hat linear abhängige Spalten

Damit lässt sich $$Cov(c,\nu) = \beta \cdot \left(J^\top J\right)^{-1}$$

nicht bilden.

Optimale Versuchsplanung



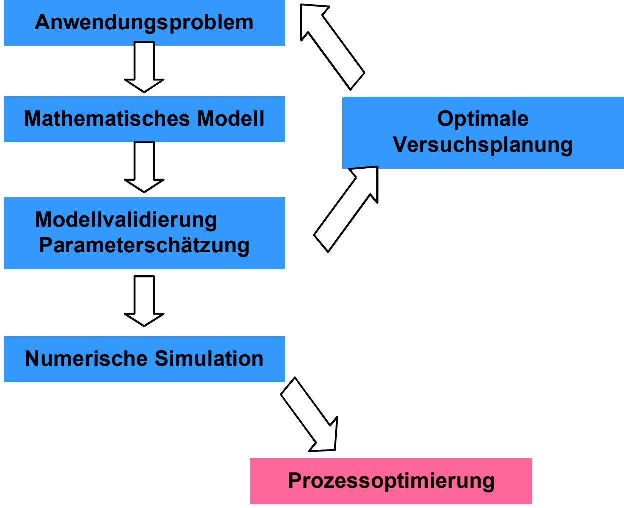
Finde Versuchsbedingungen, sodass

$$Cov(c,\nu) = \beta \cdot \left(J^\top J\right)^{-1}$$

möglichst „klein“

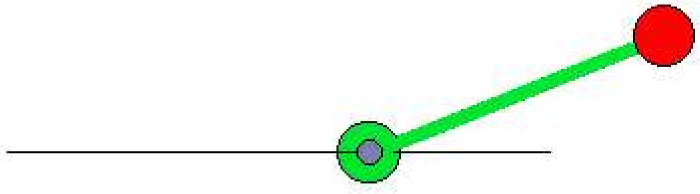
Hier:

- Messungen in 2D
- Finde z.B. x(0) so, dass spur(Cov) möglichst klein



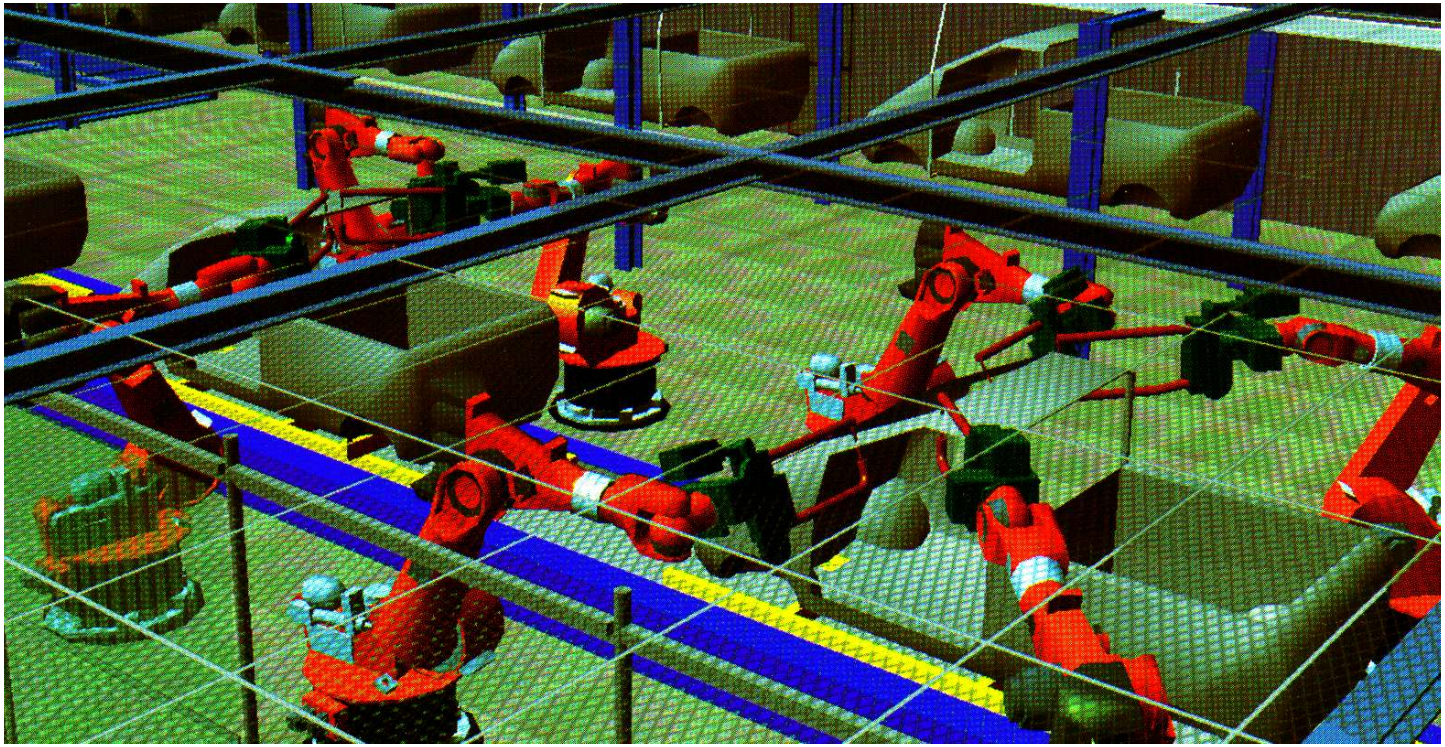
Prozessoptimierung

z.B.: Regler



- Führe eine Auslenkung möglichst schnell zur Ruhe
- Bringe Masse von A nach B und starte und ende in Ruhe

Bahnplanung von Robotern



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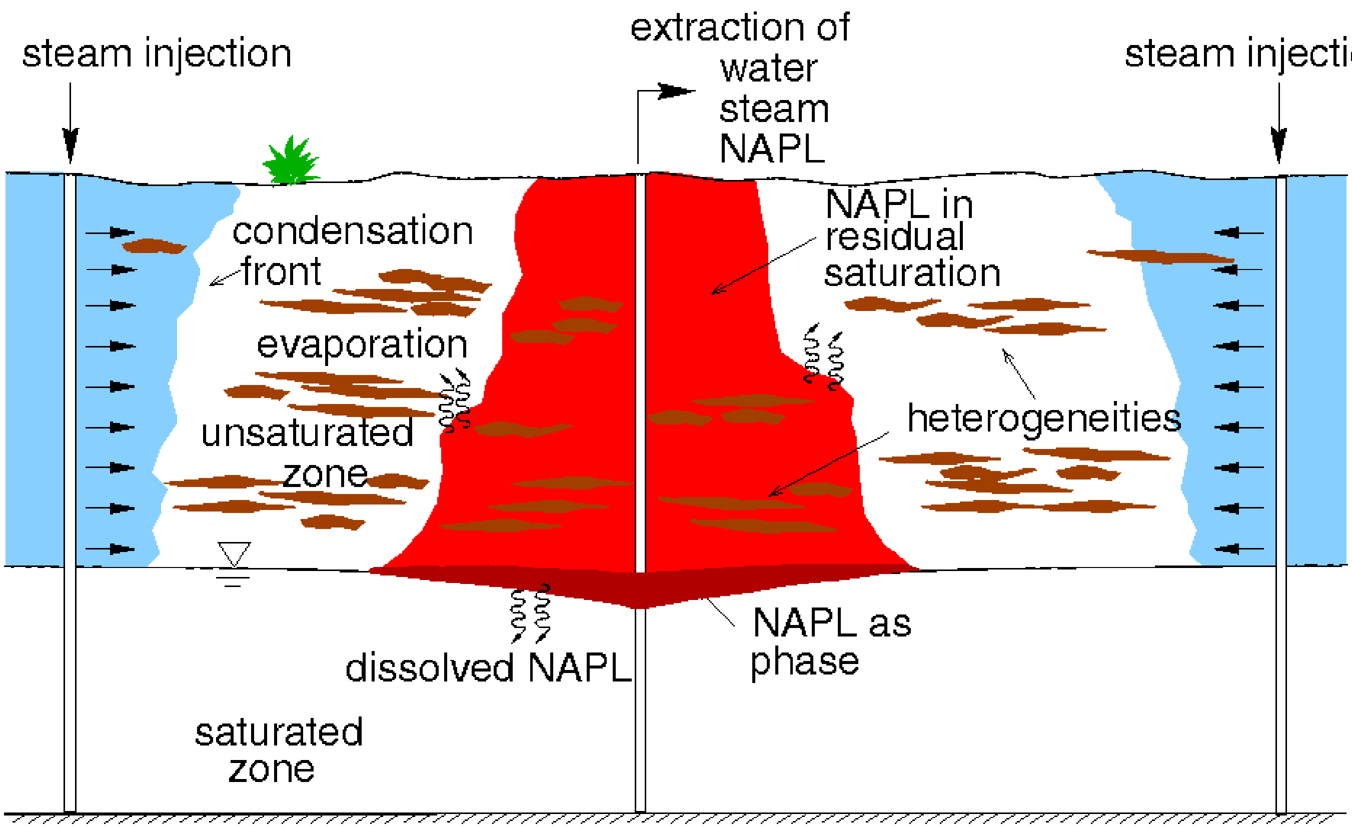
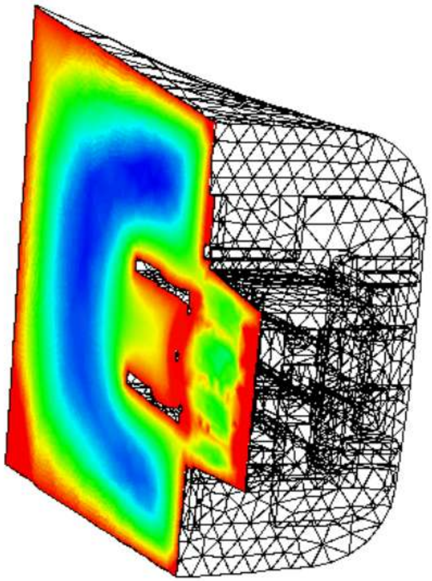
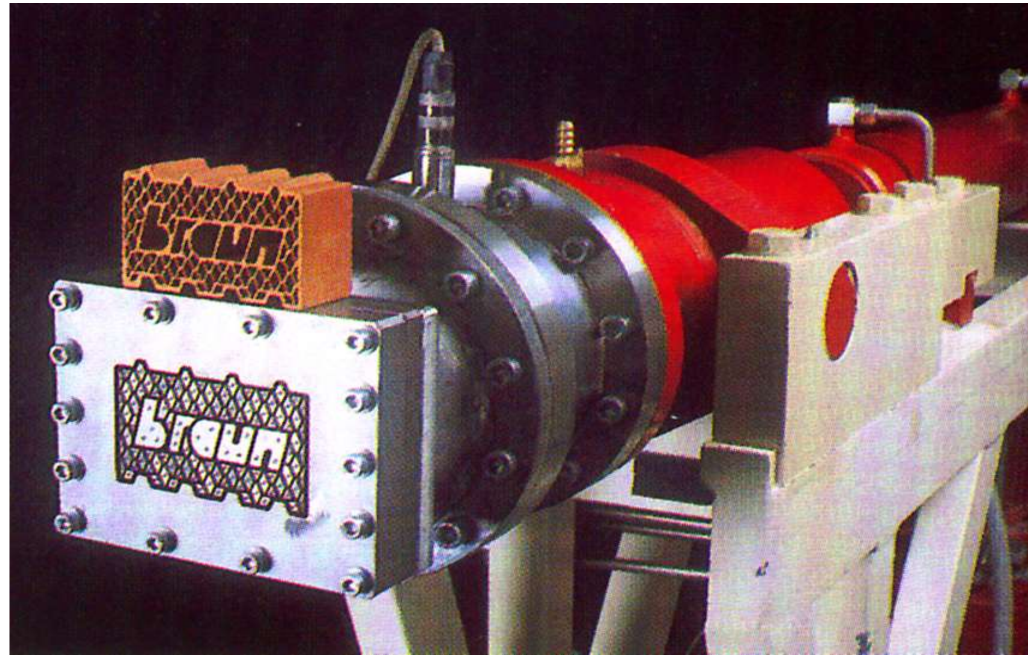
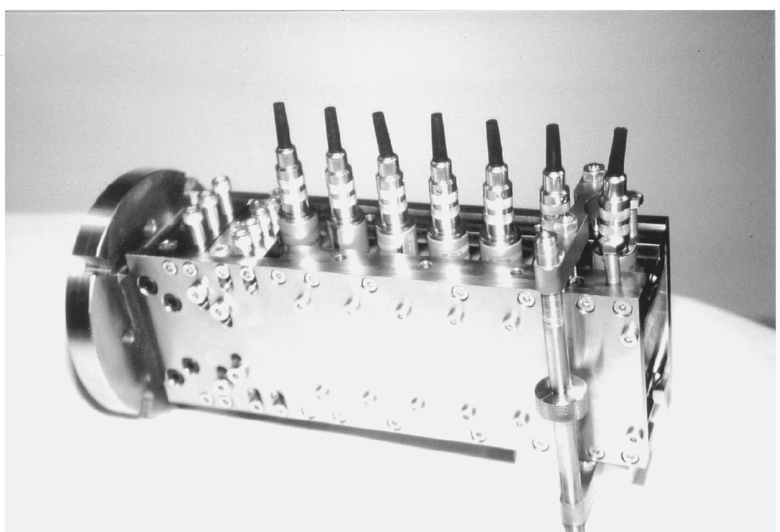
Inverse Probleme bei Mehrphasen



Pastenströmung







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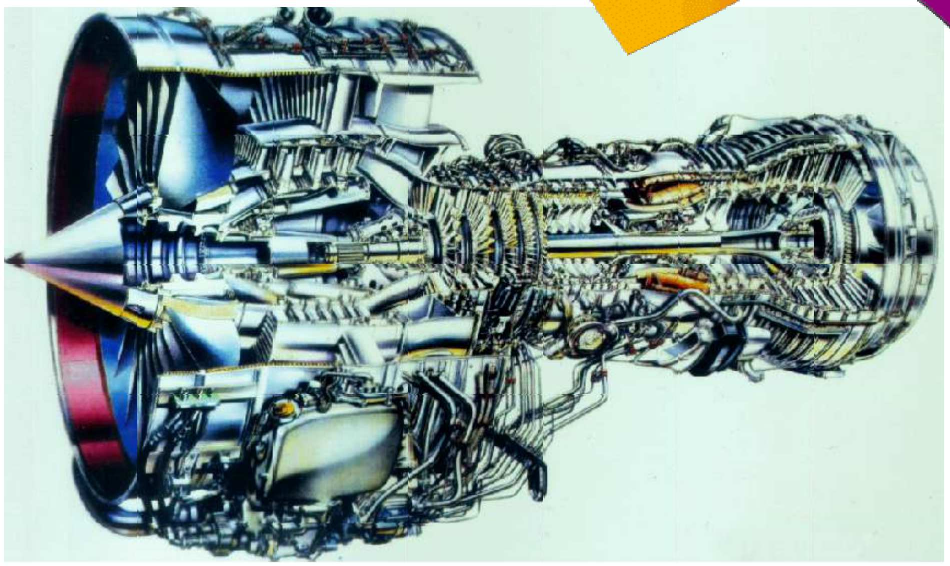
Turbinenschaufeloptimierung



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Aerodynamische Optimierung