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Bero Roos

# KERSTAN'S METHOD FOR COMPOUND POISSON APPROXIMATION

By Bero Roos

*Universität Hamburg*

We consider the approximation of the distribution of the sum of independent but not necessarily identically distributed random variables by a compound Poisson distribution and also by a finite signed measure of higher accuracy. Using Kerstan's method, some new bounds for the total variation distance are presented. Recently, several authors had difficulties applying Stein's method to the problem given. For instance, Barbour, Chen, and Loh (1992) used this method in the case of random variables on the non-negative integers. Under additional assumptions, they obtained some bounds for the total variation distance containing an undesirable log-term. In the present paper, we shall show that Kerstan's approach works without such restrictions and yields bounds without log-terms.

**1. Introduction.** In this paper, we consider the sum $S_n$ of $n \in \mathbf{N} = \{1, 2, \ldots\}$ independent random variables $X_1, \ldots, X_n$ with values in $\mathbf{R}$ and

$$p_i = P(X_i \neq 0), \qquad Q_i(B) = P(X_i \in B \mid X_i \neq 0), \qquad (i \in \{1, \ldots, n\}),$$

for Borel measurable sets $B \subseteq \mathbf{R} \setminus \{0\}$, and $\lambda = \sum_{i=1}^n p_i > 0$. We shall always assume that the condition

$$(A) \qquad\qquad Q_i = \sum_{r=1}^{\infty} q_{i,r} \, U_r, \qquad (i \in \{1, \ldots, n\})$$

is valid, where, for all $i$ and $r$, $q_{i,r} \in [0, 1]$ such that $\sum_{r=1}^{\infty} q_{i,r} = 1$ and the $U_r$ are probability measures concentrated on $\mathbf{R} \setminus \{0\}$. Clearly, condition (A) can trivially be fulfilled, if we let $U_r = Q_r$ for all $r \in \{1, \ldots, n\}$ and $q_{i,r} = \delta_{i,r}$ be the Kronecker symbol. An important non-trivial example is the following:

Example 1. If $\mathbf{R} \setminus \{0\}$ can be decomposed into pairwise disjoint measurable sets $A_r$, $(r \in \mathbf{N})$ and if, for fixed $r \in \mathbf{N}$, the conditional distributions $P(X_i \in \cdot \mid X_i \in A_r)$ are independent of $i$ and denoted by $U_r$, then condition (A) is valid

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with $q_{i,r} = P(X_i \in A_r \mid X_i \neq 0)$ for all $i$ and $r$. One of the simplest non-trivial example is the one with $U_r$ being the Dirac measure $\epsilon_{x_r}$ at point $x_r \in \mathbf{R} \setminus \{0\}$, $(r \in \mathbf{N})$.

Later we give a further example, where all the $Q_i$ are possibly different exponential distributions, (see Example 2). Sometimes we shall assume the finiteness of

$$\mu_i = \sum_{r=1}^{\infty} r \, q_{i,r}, \qquad (i \in \{1, \ldots, n\}).$$

Further, we set

$$q_r = \frac{1}{\lambda} \sum_{i=1}^{n} p_i \, q_{i,r}, \quad (r \in \mathbf{N}), \qquad \nu_i = \sum_{r=1}^{\infty} \frac{q_{i,r}^2}{q_r}, \quad (i \in \{1, \ldots, n\}).$$

Here, we define, for $r \in \mathbf{N}$, $q_{i,r}^2/q_r = 0$ whenever $q_r = 0$. Note that, if $p_i > 0$, then $\nu_i \leq \lambda/p_i$ is finite.

An important interpretation of the above setting comes from risk theory and is called the individual model: Here we consider a portfolio with $n$ policies, producing the non-negative risks $X_1, \ldots, X_n$, the so-called individual claim amounts. The probability that risk $i$ produces a claim is denoted by $p_i$, and $Q_i$ is the conditional distribution of the claim in risk $i$, given that a claim occurs in risk $i$. Further, $S_n$ is the aggregate claim in the individual model. Frequently the distribution $\mathcal{L}(S_n)$ of $S_n$ is quite involved and should be approximated by a simpler distribution. Since the $p_i$ are often assumed to be small, the compound Poisson distribution

$$\mathrm{CPo}(\lambda, \, Q) = \sum_{k=0}^{\infty} \pi(k, \, \lambda) \, Q^{*k}$$

with $\pi(k, \, \lambda) = e^{-\lambda} \, \lambda^k / k!$, $(k \in \mathbf{Z}_+ = \{0, 1, 2, \ldots\})$ is a convenient candidate, where

$$Q = \frac{1}{\lambda} \sum_{i=1}^{n} p_i \, Q_i = \sum_{r=1}^{\infty} q_r \, U_r,$$

$Q^{*k}$, $(k \in \mathbf{N})$ denotes the $k$-fold convolution of $Q$ with itself, and $Q^{*0} = \epsilon_0$. In the terminology of risk theory, $\mathrm{CPo}(\lambda, \, Q)$ is called the aggregate claims distribution in the collective model. It is well-known that $\mathrm{CPo}(\lambda, \, Q)$ can be obtained as the distribution of a sum $S_n'$, which is derived from $S_n$ by replacing $X_i$ with a random sum $\sum_{j=1}^{N_i} Y_{i,j}$, where all $N_i, Y_{i,j}$ are independent, $N_i$ is Poisson distributed with unit mean, and $Y_{i,j}$ has the same distribution as $X_i$. For general works on mathematical risk theory, we refer the reader, for example, to the books by Gerber (1979) or by Hipp and Michel (1990). Note that, below, we do not need to assume that the $X_i$ are non-negative.

In what follows, we are concerned with the accuracy of the approximation of $\mathcal{L}(S_n)$ by $\mathrm{CPo}(\lambda,\,Q)$ and also by a related finite signed measure of second order. As measures of accuracy, we consider the total variation distance and the Kolmogorov metric

$$d_{\mathrm{TV}}(R_1,\,R_2) = \sup_{A} |R_1(A) - R_2(A)|,$$

$$d_{\mathrm{KM}}(R_1,\,R_2) = \sup_{x \in \mathbf{R}} |F_1(x) - F_2(x)|$$

between two finite signed measures $R_1$ and $R_2$ on $\mathbf{R}$ with distribution functions $F_1$ and $F_2$. Here the $\sup_A$ is taken over all Borel measurable sets $A \subseteq \mathbf{R}$. In the case that $R_i$ is the distribution of a random variable $Z_i$, we also write $d_{\mathrm{TV}}(Z_1,\,Z_2)$ and $d_{\mathrm{KM}}(Z_1,\,Z_2)$ for the respective distances between $R_1$ and $R_2$. We often use the abbreviation

$$d_\tau = d_{\mathrm{TV}}(\mathcal{L}(S_n),\,\mathrm{CPo}(\lambda,\,Q)), \qquad d_\kappa = d_{\mathrm{KM}}(\mathcal{L}(S_n),\,\mathrm{CPo}(\lambda,\,Q)).$$

As mentioned above, we always suppose that condition (A) is valid. Furthermore, we sometimes suppose that some of the following conditions are true:

(B)             $U_r = U^{*r}$     for all $r \in \mathbf{N},$       $(U = U_1),$

(C)         $U_r = \epsilon_r = \epsilon_1^{*r}$     for all $r \in \mathbf{N},$

(D)     $r q_r \geq (r+1) q_{r+1}$     for all $r \in \mathbf{N}.$

We use a method originally due to Kerstan (1964), which was refined in Daley and Vere-Jones (1988, pp. 297–299), Witte (1990), and Roos (1996, Kapitel 8; 1999a). In the latter paper, it was used to remove an undesirable log-term, which appeared in Barbour's (1988) upper bound for the total variation distance between the generalized multinomial distribution and a multivariate Poisson law with independent components and the same mean vector. The present paper shows further advantages of Kerstan's approach over Stein's method in the compound Poisson approximation. However, it should be mentioned that, in contrast to Kerstan's approach, Stein's method remains easily applicable in the case of dependent random variables.

The paper is structured as follows: The following section is devoted to a review of known results under different conditions. In Section 3, we present our first- and second-order results, the proofs of which are contained in Section 4.

## 2. Review of some known results

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2.1. *The Khintchine-Doeblin inequality.* The first inequality known for the total variation distance is generally valid and is essentially due to Khintchine (1933) and Doeblin (1939) (see also Le Cam (1960, p. 1183)). It reads as

$$(1) \qquad d_\tau \leq \sum_{i=1}^n p_i^2 =: \lambda_2.$$

If we consider large $n$, this bound is often greater than one and therefore useless, since $d_\tau$ is always bounded by one. However, it turns out that, in special cases, the order can be improved (see below).

2.2. *The case $Q_1 = \ldots = Q_n = \epsilon_1$.* If $Q_1 = \ldots = Q_n = \epsilon_1$ is the Dirac measure at point 1, bounds of better order are possible (for example, see Prohorov (1953) for the binomial case, Le Cam (1960), Kerstan (1964), Barbour and Hall (1984), Deheuvels and Pfeifer (1986), Witte (1990), and Roos (1999b, 2001)). One of the most remarkable results is due to Barbour and Hall. By using Stein's method, they proved in their Theorems 1 and 2 that, in the case above,

$$(2) \qquad \frac{\lambda_2}{32} \min\left\{\frac{1}{\lambda}, 1\right\} \leq d_\tau \leq \lambda_2 \frac{1 - e^{-\lambda}}{\lambda} \leq \lambda_2 \min\left\{\frac{1}{\lambda}, 1\right\};$$

here, $d_\tau$ and $\lambda_2 \min\{\lambda^{-1}, 1\}$ are of the same order. It is easy to show that, generally, the constants of the upper bound in (2) cannot be reduced. But under further assumptions, more can be done: In Roos (1999b, formula (32)), we gave the following sharpening of a result of Deheuvels and Pfeifer (1986):

$$(3) \qquad \left|\frac{\sqrt{2\pi e}}{\theta} d_\tau - 1\right| \leq C_1 \min\left\{1, \frac{1}{\sqrt{\lambda}} + \theta\right\}, \qquad \theta = \frac{\lambda_2}{\lambda},$$

where $C_1 \in (0, \infty)$ is an absolute constant. In the binomial case, this was shown by Prohorov (1953, Theorem 2), (see also Barbour, Holst, and Janson (1992, p. 2)). In particular, here we have $d_\tau \sim \theta/\sqrt{2\pi e}$, as $\lambda \to \infty$ and $\theta \to 0$. In other words, if $\lambda \geq a > 0$ is large and $\theta \leq \epsilon \leq 1$ is small, then $d_\tau \leq C_2(a, \epsilon)\,\theta$ with $C_2(a, \epsilon)$ being nearly equal to $1/\sqrt{2\pi e}$, which, in turn is asymptotically optimal for $a \to \infty$ and $\epsilon \to 0$. There arose the problem of an inequality containing a sharp constant if we only assume that $\theta$ is small. In Roos (2001, formula (10)), we gave such a bound, which reads as follows:

$$(4) \qquad d_\tau \leq \left(\frac{3}{4e} + \frac{7\sqrt{\theta}(3 - 2\sqrt{\theta})}{6(1 - \sqrt{\theta})^2}\right)\theta.$$

Further, we showed that the "$\leq$" in (4) can be replaced with "$\sim$", if $\theta \to 0$ and $\lambda \to 1$, which indicates that the constant $3/(4e)$ in (4) is best possible. In particular, if we assume only small $\theta \leq \epsilon \leq 1$, we have an inequality of the form $d_\tau \leq C_3(\epsilon)\,\theta$ with $C_3(\epsilon)$ being nearly equal to $3/(4e)$, which, in turn, is the asymptotical optimal

constant for $\epsilon \to 0$. Note that, comparing the constants in (3) and (4), we see that $1/\sqrt{2\pi e} \approx 0.24$ differs not much from $3/(4e) \approx 0.28$. It should be mentioned that, for the Kolmogorov metric and other distances, results similar to (3) and (4) can be found in the papers cited above.

2.3. *Results under further conditions.* From a simple observation made by Le Cam (1965, p. 187) and later used by Michel (1987, p. 167), it follows that the upper bound in (2) remains valid in the case $Q_1 = \ldots = Q_n \neq \epsilon_1$. But if the $Q_i$ are arbitrary and possibly unequal, an upper bound for $d_\tau$ independent of the $Q_i$ cannot have order better than $\min\{\lambda_2, 1\}$ (see Le Cam (1965, p. 188) and Zaitsev (1989, Remark 1.1)). Barbour and Utev (1999, Example 1.4) observed that this statement even holds in the presence of the condition (C). However, for the weaker Kolmogorov metric, more can be done, as shown by Le Cam (1965) and Zaitsev (1983). We only cite Zaitsev's result: In the general case, the inequality

$$(5) \qquad\qquad\qquad d_\kappa \le C_4 \, p_{\max}$$

holds, where $C_4 \in (0, \infty)$ is an absolute constant and $p_{\max} = \max_{1 \le i \le n} p_i$. Note that, since $\theta = \lambda_2/\lambda \le p_{\max}$, the upper bound in (5) has weaker order than $\theta$. To get bounds for $d_\tau$ of better order than $\min\{\lambda_2, 1\}$, Barbour, Chen, and Loh (1992, Theorem 5) considered the case that the conditions (C) and (D) are satisfied. In this context, we have $q_r = Q(\{r\})$ for $r \in \mathbf{N}$, $Q_1, \ldots, Q_n$ are concentrated on $\mathbf{N}$, and $\mu_i$ is the mean of $Q_i$. In particular, their results (see also Barbour and Utev (1999, Proposition 1.5)) imply that, in the case above, the inequality

$$(6) \qquad\qquad\qquad d_\tau \le H(\lambda, Q) \sum_{i=1}^{n} p_i^2 \, \mu_i^2$$

holds and $H(\lambda, Q)$ is a quantity, which was derived with the help of Stein's method and satisfies

$$(7) \quad H(\lambda, Q) \le \min\left\{1, \frac{1}{\lambda\,(q_1 - 2q_2)}\left(\frac{1}{4\,\lambda\,(q_1 - 2q_2)} + \log^+(2\,\lambda\,(q_1 - 2q_2))\right)\right\}.$$

Therefore, if the conditions (C) and (D) hold and if the $Q_i$ have bounded means and $q_1 - 2q_2$ is bounded away from zero, then $d_\tau$ is of order $O(\lambda_2 \min\{\lambda^{-1}(1 + \log^+ \lambda), 1\})$. Barbour and Xia (1999, Theorem 2.5) [see also Barbour and Chryssaphinou (2001, formula (2.21))] have shown that, if condition (C) is valid and if

$$\vartheta = \sum_{r=2}^{\infty} \frac{r(r-1)q_r}{\mu} < \frac{1}{2}, \qquad \mu = \sum_{r=1}^{\infty} r q_r,$$

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then

$$(8) \qquad\qquad H(\lambda, Q) \leq \frac{1}{(1 - 2\vartheta)\lambda\mu}.$$

In Theorem 2 of the present paper, we will show substantial improvements of such inequalities under weaker assumptions. In particular, this theorem implies that, in the presence of condition (B), we have

$$(9) \qquad\qquad d_\tau = O\Big( \sum_{i=1}^{n} p_i^2 \min\Big\{ \frac{\mu_i^2}{\lambda q_1}, \frac{\nu_i}{\lambda}, 1 \Big\} \Big)$$

and that, if additionally condition (D) is valid, the "$q_1$" in the denominator of the right-hand side of (9) can be removed. Note that $(1 - 2\vartheta)\mu \leq 2q_1 - \mu \leq q_1$, which shows that, in fact, the order in (9) is better than that of (6) together with (8). Barbour and Utev (1999, p. 93) pointed out that, if condition (D) is not satisfied, the best inequality known would be (see also Barbour, Chen, and Loh (1992, Theorem 4))

$$(10) \qquad\qquad H(\lambda, Q) \leq e^\lambda \min\Big\{ \frac{1}{\lambda q_1}, 1 \Big\};$$

note that the right-hand side of (10) is bounded away from zero, which shows that, under the present assumptions, the method of Barbour, Chen, and Loh (1992) leads to unsatisfying results. For the Kolmogorov metric, Barbour and Xia (2000, Proposition 1.1) proved that, if both conditions (C) and (D) are satisfied, then

$$(11) \qquad\qquad d_\kappa \leq \min\Big\{ \frac{1}{\lambda\, q_1 + 1}, \frac{1}{2} \Big\} \sum_{i=1}^{n} p_i^2 \mu_i^2.$$

Since $d_\kappa \leq d_\tau$, our Theorem 2 leads to a result better than (11), if we take into account only the order of the bounds.

See Barbour and Utev (1998, 1999), for refinements of the method given in Barbour, Chen, and Loh (1992) and for other somewhat complicated estimates of $d_\tau$ and $d_\kappa$. Further contributions on compound Poisson approximation, came, for example, from Hipp (1985), Čekanavičius (1997, 1998), and Vellaisamy and Chaudhuri (1999).

**3. Main results.** Recall that, as mentioned above, we always assume that condition (A) holds.

3.1. *First-order results.* The following theorems concern the total variation distance $d_\tau$. Some notation is necessary. For $x \in (0, \infty)$ and $z \in \mathbf{C}$, let

$$g(z) = 2\,\frac{e^z}{z^2}\,(e^{-z} - 1 + z),$$

$$\alpha_1(x) = \sum_{i=1}^n g(2p_i)\,p_i^2 \min\left\{\frac{x\,\nu_i}{\lambda},\,1\right\},$$

$$\beta_1 = \sum_{i=1}^n p_i^2 \min\left\{\frac{\nu_i}{\lambda},\,1\right\}.$$

The following bounds are direct consequences of Theorem 1 in Roos (1999a) and are valid without assuming further constraints.

THEOREM 1. *We have*

(12)
$$d_\tau \le \frac{\alpha_1(2^{-3/2})}{1 - 2\,e\,\alpha_1(2^{-3/2})}, \qquad if \quad \alpha_1(2^{-3/2}) < \frac{1}{2e};$$

(13)
$$d_\tau \le 8.8\,\beta_1.$$

REMARK 1.

(a) Theorem 1 also holds in more general situations, where, for example, the $U_r$ are multidimensional (or even more general) measures. All what we need for a proof is the finite operator norm applicable to convolutions of operators, see Le Cam (1965).

(b) For practical usage, (12) is often sharper than (13), which, in turn, has theoretical value.

(c) We have

$$1 \le \max_{1 \le i \le n} g(2p_i) \le g(2p_{\max}) \le g(2) \le 4.1946, \quad \text{and} \quad \lim_{\epsilon \downarrow 0} g(\epsilon) = 1.$$

(d) Unfortunately, the Khintchine-Doeblin bound (1) is not contained in the bounds of Theorem 1. From (13), it only follows that $d_\tau \le c\lambda_2$ with $c = 8.8$. But if we consider the case of small $\lambda_2$, it follows from Remark (c) that (12) contains the bound with $c \approx 1$.

The next theorem requires further notation: For $x \in (0, \infty)$, let

$$\alpha_2(x) = \sum_{i=1}^n g(2p_i)\,p_i^2 \min\left\{\frac{x\,\mu_i^2}{\lambda},\,\frac{\nu_i}{2^{3/2}\,\lambda},\,1\right\},$$

$$\beta_2(x) = \sum_{i=1}^n p_i^2 \min\left\{\frac{x\,\mu_i^2}{\lambda},\,\frac{\nu_i}{\lambda},\,1\right\}.$$

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THEOREM 2. *Let $c_1 = 3/(4e)$. In the presence of condition* (B), *we have*

$$(14) \qquad d_\tau \le \frac{\alpha_2(c_1\,q_1^{-1})}{1 - 2\,e\,\alpha_2(c_1\,q_1^{-1})}, \qquad if \quad \alpha_2(c_1\,q_1^{-1}) < \frac{1}{2e};$$

$$(15) \qquad d_\tau \le 8.8\,\beta_2(q_1^{-1}).$$

*In the presence of* (B) *and* (D),

$$(16) \qquad d_\tau \le \frac{\alpha_2(e^{-1})}{1 - 2\,e\,\alpha_2(e^{-1})}, \qquad if \quad \alpha_2(e^{-1}) < \frac{1}{2e};$$

$$(17) \qquad d_\tau \le 9.2\,\beta_2(1).$$

REMARK 2.

(a) Generally, in (14), the constant $c_1$ cannot be reduced. Indeed, if condition (C) holds, $Q_1 = \ldots = Q_n = \epsilon_1$, $p_{\max} \to 0$, and $\lambda \to 1$, then we obtain $\alpha_2(cq_1^{-1})/(1 - 2e\alpha_2(cq_1^{-1})) \sim c\,\lambda_2$ for $c \in (0, c_1]$, and, in view of Roos (2001, Theorem 2), we see that $d_\tau \sim c_1\,\lambda_2$ (see also Subsection 2.2 of the present paper).

(b) The inequalities of Theorem 2 are substantial improvements of the bounds, which can be derived from the results of Barbour, Chen, and Loh (1992) and Barbour and Xia (1999), (see (6)–(8) above).

EXAMPLE 2. For $i \in \{1, \ldots, n\}$, let $Q_i = \mathcal{E}(t_i)$ be an exponential distribution with parameter $t_i \in (0, \infty)$. Let $a > t_{\max} = \max_{1 \le i \le n} t_i$ be arbitrary. Then it is easily shown that conditions (A) and (B) are satisfied with

$$q_{i,r} = (1 - b_i)^{r-1} b_i, \qquad b_i = \frac{t_i}{a}, \qquad U_r = (\mathcal{E}(a))^{*r}.$$

Indeed, the characteristic functions of $Q_i$ and $\sum_{r=1}^{\infty} q_{i,r}(\mathcal{E}(a))^{*r}$ coincide. Using (15), it now follows that, if $t_{\max}$ is bounded and $t_{\min} = \min_{1 \le i \le n} t_i$ is bounded away from zero, then $d_\tau = O(\lambda_2 \min\{\lambda^{-1}, 1\})$. In particular, by letting $a \downarrow t_{\max}$, we obtain

$$(18) \quad d_\tau \le 8.8 \sum_{i=1}^{n} p_i^2 \min\left\{\frac{t_{\max}^3}{t_i^2 \sum_{j=1}^{n} p_j t_j}, 1\right\} \le 8.8\,\lambda_2 \min\left\{\frac{1}{\lambda}\left(\frac{t_{\max}}{t_{\min}}\right)^3, 1\right\}.$$

If $2t_{\min} > t_{\max}$, then, as is easily shown, condition (D) is satisfied and (17) leads to

$$d_\tau \le 9.2 \sum_{i=1}^{n} p_i^2 \min\left\{\frac{t_{\max}^2}{\lambda\,t_i^2}, 1\right\} \le 9.2\,\lambda_2 \min\left\{\frac{1}{\lambda}\left(\frac{t_{\max}}{t_{\min}}\right)^2, 1\right\},$$

which is, under the present assumptions, of the same order as (18). Note that, in this example, it is difficult to evaluate the $\nu_i$, so that we did not use them in the bounds above.

3.2. *Second-order results.* The following theorems are devoted to the second-order approximation of $\mathcal{L}(S_n)$ by the finite signed measure

$$(19) \qquad \mathrm{CPo}_2(\lambda,\, Q) = \left(\epsilon_0 - \frac{1}{2}\sum_{i=1}^n p_i^2 (Q_i - \epsilon_0)^{*2}\right) * \mathrm{CPo}(\lambda,\, Q).$$

We shall give bound for

$$d_\tau' := d_{\mathrm{TV}}(\mathcal{L}(S_n),\, \mathrm{CPo}_2(\lambda,\, Q)).$$

Let

$$h(z) = \frac{3\,(g(z) - 1)}{2\,z}, \qquad (z \in \mathbf{C})$$

$$\gamma_1 = \sum_{i=1}^n h(2p_i)\, p_i^3\, \min\left\{0.46\left(\frac{\nu_i}{\lambda}\right)^{3/2}, 1\right\}.$$

The next theorem is generally valid and follows from Theorem 2 in Roos (1999a).

THEOREM 3.    *We have*

$$d_\tau' \leq \frac{4}{3}\gamma_1 + (\alpha_1(1))^2\left(1 + \frac{0.82\,\alpha_1(1)}{1 - 2^{-1/2}\alpha_1(1)\,e}\right),$$

*where we assume that* $\alpha_1(1) < 2^{1/2}e^{-1}$.

For $x \in (0,\, \infty)$, let

$$\gamma_2(x) = \sum_{i=1}^n h(2p_i)\, p_i^3\, \min\left\{\frac{x\,\mu_i^3}{\lambda^{3/2}}, 0.46\left(\frac{\nu_i}{\lambda}\right)^{3/2}, 1\right\}.$$

THEOREM 4.    *If condition* (B) *is valid, then*

$$(20) \qquad d_\tau' \leq \frac{4}{3}\gamma_2(0.31\,q_1^{-3/2}) + \frac{4(\alpha_2(c_1 q_1^{-1}))^2}{1 - 2\,e\,\alpha_2(c_1 q_1^{-1})},$$

*where we suppose that* $\alpha_2(c_1 q_1^{-1}) < 1/(2e)$ *and* $c_1$ *is defined in Theorem 2. In the presence of* (B) *and* (D),

$$(21) \qquad d_\tau' \leq \frac{4}{3}\gamma_2(0.41) + \frac{4.2\,(\alpha_2(e^{-1}))^2}{1 - 2\,e\,\alpha_2(e^{-1})}, \qquad if \quad \alpha_2(e^{-1}) < \frac{1}{2e}.$$

The estimates in Subsection 4.2 below and an inequality in Roos (1999a, p. 132) yield some bounds for the total variation norm (for a definition, see Subsection 4.1) of the finite signed measure $\mathrm{CPo}_2(\lambda,\, Q)$:

Lemma 1. *In the general case,*

$$\|\mathrm{CPo}_2(\lambda,\,Q)\| \le 1 + 2\beta_1.$$

*If condition* (B) *is valid, then*

$$\|\mathrm{CPo}_2(\lambda,\,Q)\| \le 1 + 2\beta_2(q_1^{-1}).$$

*If conditions* (B) *and* (C) *are satisfied, then*

$$\|\mathrm{CPo}_2(\lambda,\,Q)\| \le 1 + 2\beta_2(1).$$

Remark 3. With the help of Lemma 1, one can remove the singularities in the upper bounds of Theorems 3 and 4. For example, in the context of Theorem 3, an absolute constant $c \in (0, \infty)$ can easily be found such that $d'_\tau \le (4/3)\gamma_1 + c(\alpha_1(1))^2$. Similar statements are possible concerning the bounds in Theorem 4. Unfortunately, the constants involved are quite large, so that we do not state them explicitly. Note that Čekanavičius (1998, proof of Corollary 3.1) has shown that, in the general case,

$$d'_\tau \le 2\lambda_2^2 + \frac{8}{3}\sum_{i=1}^{n} p_i^3.$$

It is easily seen that a bound of this order is also contained in the estimates of Theorems 3 and 4.

Using the bounds for $d'_\tau$ and a separate consideration of the leading term in Kerstan's expansion (see below), it is possible to derive a further interesting first-order result:

Theorem 5. *In the presence of* (B) *and* (D),

$$(22) \quad d_\tau \le d'_\tau + \sum_{i=1}^{n} p_i^2 \min\left\{\mu_i^2\,\mathrm{E}\binom{Y+2}{2}^{-1/2},\ \mu_i^2\,e^{-\min\{\lambda,\,2q_1^{-1}\}},\ \frac{\nu_i}{2^{3/2}\,\lambda},\ 1\right\},$$

*where $Y$ is a random variable with distribution* $\mathrm{CPo}(\lambda, \sum_{r=1}^{\infty} q_r \epsilon_r)$.

The bound in (22) can be further estimated, by using the following simple inequalities:

$$(23) \qquad \mathrm{E}\binom{Y+2}{2}^{-1/2} \le \sqrt{2}\,\mathrm{E}\frac{1}{1+Y} \le \sqrt{2}\,\frac{1-e^{-\lambda}}{\lambda}.$$

## 4. Proofs

4.1. *Kerstan's method.* We use the following notation. Products and powers of finite signed measures on $\mathbf{R}$ are understood in the sense of convolution. For a finite signed measure $R$ on $\mathbf{R}$ with Hahn-Jordan decomposition $R = R_+ - R_-$, we define the total variation norm of $R$ by $\|R\| = (R_+ + R_-)(\mathbf{R})$. We shall repeatedly use the easy inequality

$$\|R_1 R_2\| \leq \|R_1\| \, \|R_2\|$$

for two finite signed measures $R_1$ and $R_2$. The proofs of our results are based on a slightly modification of Kerstan's expansion (cf. Roos (1999a)), which reads as

$$\mathcal{L}(S_n) - \mathrm{CPo}(\lambda, Q) = \left( \prod_{i=1}^{n} (L_i + \epsilon_0) - \epsilon_0 \right) \mathrm{CPo}(\lambda, Q)$$

$$(24) \qquad = \sum_{j=1}^{n} \sum_{1 \leq i(1) < \ldots < i(j) \leq n} \prod_{s=1}^{j} \left( L_{i(s)} \, \mathrm{CPo}\!\left(\frac{\lambda}{j}, Q\right) \right),$$

where, for $i \in \{1, \ldots, n\}$,

$$L_i = \left( \epsilon_0 + p_i (Q_i - \epsilon_0) \right) \exp\left( -p_i (Q_i - \epsilon_0) \right) - \epsilon_0$$

$$(25) \qquad = -\frac{p_i^2}{2} (Q_i - \epsilon_0)^2 \, g(p_i(\epsilon_0 - Q_i)).$$

Applying the polynomial theorem, we are now led to

$$d_\tau = \frac{1}{2} \|\mathcal{L}(S_n) - \mathrm{CPo}(\lambda, Q)\|$$

$$\leq \frac{1}{2} \sum_{j=1}^{n} \sum_{1 \leq i(1) < \ldots < i(j) \leq n} \prod_{s=1}^{j} \left\| L_{i(s)} \, \mathrm{CPo}\!\left(\frac{\lambda}{j}, Q\right) \right\|$$

$$(26) \qquad \leq \frac{1}{2} \sum_{j=1}^{n} \frac{1}{j!} \left( \sum_{i=1}^{n} \left\| L_i \, \mathrm{CPo}\!\left(\frac{\lambda}{j}, Q\right) \right\| \right)^j.$$

In what follows, we are concerned with suitable norm estimates.

4.2. *Norm estimates*

LEMMA 2. *Let $t \in (0, \infty)$, $i \in \{1, \ldots, n\}$, and $c_1$ be as in Theorem 2. Then, in the presence of condition* (B), *we have*

$$(27) \qquad \|(U - \epsilon_0) \, \mathrm{CPo}(t, Q)\| \leq \sqrt{\frac{2}{t \, q_1 \, e}},$$

$$(28) \qquad \|(U - \epsilon_0)^2 \, \mathrm{CPo}(t, Q)\| \leq \frac{3}{t \, q_1 \, e},$$

$$(29) \qquad \|(U - \epsilon_0)^3 \operatorname{CPo}(t, Q)\| \leq \frac{3}{\sqrt{2}} \left( \frac{3}{t \, q_1 \, e} \right)^{3/2},$$

$$(30) \qquad \|(Q_i - \epsilon_0)^2 \operatorname{CPo}(t, Q)\| \leq \frac{\sqrt{2} \, \nu_i}{t},$$

$$(31) \qquad \|(Q_i - \epsilon_0)^3 \operatorname{CPo}(t, Q)\| \leq \frac{1}{\sqrt{2}} \left( \frac{3 \, \nu_i}{t} \right)^{3/2},$$

$$(32) \qquad \|L_i \operatorname{CPo}(t, Q)\| \leq 2 \, g(2p_i) \, p_i^2 \, \min \left\{ \frac{c_1 \, \mu_i^2}{t \, q_1}, \frac{\nu_i}{2^{3/2} \, t}, 1 \right\}.$$

*If additionally* $t \geq p_i \sup_{r \in \mathbf{N}} q_{i,r}/q_r$, *then*

$$(33) \qquad \|L_i \operatorname{CPo}(t, Q)\| \leq 2 \, p_i^2.$$

PROOF. Inequalities (27) and (28) follow from Roos (2001, formulas (27) and (29)) and the observation that, for $q_1 < 1$,

$$\operatorname{CPo}(t, Q) = \operatorname{CPo}(tq_1, U) \operatorname{CPo}(t(1 - q_1), \tilde{Q}), \qquad \tilde{Q} = \sum_{m=2}^{\infty} \frac{q_m}{1 - q_1} U^m.$$

The proof of (29) is easily done with the help of (27), (28) and the inequality

$$\|(U - \epsilon_0)^3 \operatorname{CPo}(t, Q)\| \leq \left\| (U - \epsilon_0) \operatorname{CPo}\left( \frac{t}{3}, Q \right) \right\| \left\| (U - \epsilon_0)^2 \operatorname{CPo}\left( \frac{2t}{3}, Q \right) \right\|.$$

The bounds (30) and (31) follow from Roos (1999a, formulas (19) and (26)) and the simple fact that, for $j \in \mathbf{Z}_+$,

$$\|(Q_i - \epsilon_0)^j \operatorname{CPo}(t, Q)\| = \lim_{k \to \infty} \left\| \left( \sum_{r=1}^{k} q_{i,r} (U_r - \epsilon_0) \right)^j \operatorname{CPo}\left( t^{(k)}, Q^{(k)} \right) \right\|,$$

where

$$(34) \qquad t^{(k)} = t \sum_{r=1}^{k} q_r, \qquad Q^{(k)} = \sum_{r=1}^{k} \frac{t \, q_r}{t^{(k)}} U_r.$$

Inequality (32) is an immediate consequence of (25), (28), (30), and the representation

$$(35) \qquad Q_i - \epsilon_0 = \left( \sum_{k=0}^{\infty} \sum_{r=k+1}^{\infty} q_{i,r} \, U^k \right) (U - \epsilon_0).$$

The proof of (33) is similar to the one of (23) in Roos (1999a) and is therefore omitted. $\qquad \square$

REMARK 4. (a) If $U = Q = \epsilon_1$, $\operatorname{CPo}(t, Q)$ is a Poisson distribution with mean $t$ and it follows from Proposition 4 of Roos (1999b) that, in this context, we have,

for $t \to \infty$ and fixed $j \in \mathbf{Z}_+$,

$$\|(U - \epsilon_0)^j \operatorname{CPo}(t, Q)\| \sim \frac{1}{t^{j/2}} \int_{\mathbf{R}} \left| \frac{1}{\sqrt{2\pi}} \frac{d^j}{dx^j} e^{-x^2/2} \right| dx.$$

In particular, here the norm term is of order $O(t^{-j/2})$ for $t \to \infty$. But in the general case, this need not be true for $j \in \mathbf{N}$. For example, if $U = \epsilon_1$ and $Q = \epsilon_{j+1}$, we easily get $\|(U - \epsilon_0)^j \operatorname{CPo}(t, Q)\| = 2^j$. This explains, why the "$q_1$" appears in the denominator of some upper bounds given in Lemma 2.

(b) From (33), it follows that $\|L_i \operatorname{CPo}(\lambda, Q)\| \leq 2 p_i^2$.

In the presence of the conditions (B) and (D), we shall give norm estimates better than (27)–(29). For the proof, we need the following lemma.

LEMMA 3.    *Let $t \in (0, \infty)$. If the conditions* (B) *and* (D) *are satisfied, then, for all $\eta \in (0, 1)$, there exists a distribution $R_\eta$ on $\mathbf{R}$ such that the decomposition*

$$\operatorname{CPo}(t, Q) = \operatorname{CPo}(t, Q(\eta)) R_\eta, \qquad Q(\eta) = \sum_{r=1}^{\infty} q_r (\epsilon_0 + \eta(U - \epsilon_0))^r$$

*is valid.*

This lemma is essentially due to Steutel and van Harn (1979, Lemma 1.2 and Theorem 2.2), who investigated the case $U = \epsilon_1$. Under this assumption, the above property of $\operatorname{CPo}(t, Q)$ is called discrete self-decomposability. The generalization of the statement by Steutel and van Harn to arbitrary distributions $U$ is trivial.

LEMMA 4.    *Let $t \in (0, \infty)$ and $j \in \mathbf{Z}_+$. Further, let $Y$ denote a random variable with distribution $\mathcal{L}(Y) = \operatorname{CPo}(t, \sum_{r=1}^{\infty} q_r \epsilon_r)$. We assume that* (B) *and* (D) *are satisfied. Then*

(36) $$\|(U - \epsilon_0)^j \operatorname{CPo}(t, Q)\| \leq 2^j \operatorname{E} \binom{Y + j}{j}^{-1/2}.$$

PROOF.    From Lemma 3, we obtain, for all $\eta \in (0, 1)$,

$$\|(U - \epsilon_0)^j \operatorname{CPo}(t, Q)\| \leq \|(U - \epsilon_0)^j \operatorname{CPo}(t, Q(\eta))\|$$

$$\leq \operatorname{E} \left\| (U - \epsilon_0)^j (\epsilon_0 + \eta(U - \epsilon_0))^Y \right\|$$

$$\leq \frac{1}{(\eta(1 - \eta))^{j/2}} \operatorname{E} \binom{Y + j}{j}^{-1/2},$$

where the latter inequality follows from Roos (2000, formula (37)). The proof of (36) is completed by letting $\eta = 1/2$. $\square$

REMARK 5. From (36) and (23), we derive an inequality better than (28): If (B) and (D) are valid and $t \in (0, \infty)$, then

$$(37) \qquad \|(U - \epsilon_0)^2 \, \mathrm{CPo}(t, Q)\| \le 4\sqrt{2} \, \frac{1 - e^{-t}}{t}.$$

In particular, here the "$q_1$" does not appear in the denominator. In what follows, we show that (37) can be further improved. We need the following lemma due to Steutel and van Harn (1979, Lemma 1.2 and Theorem 2.3).

LEMMA 5. *Let $t \in (0, \infty)$. If the conditions* (C) *and* (D) *are satisfied, then* $\mathrm{CPo}(t, Q)$ *is discrete unimodal. This means that, if $a_m = \mathrm{CPo}(t, Q)(\{m\})$ for $m \ge -1$, then the sequence $(a_{m-1} - a_m)_{m \in \mathbf{Z}_+}$ changes sign at most once.*

LEMMA 6. *Let the conditions of Lemma 4 be valid and $i \in \{1, \dots, n\}$. Then*

$$(38) \qquad \|(U - \epsilon_0)^{j+1} \, \mathrm{CPo}(t, Q)\| \le \left( \frac{2 \, (j+1)}{t \, e} \right)^{(j+1)/2},$$

$$(39) \qquad \|L_i \, \mathrm{CPo}(t, Q)\| \le 2 \, g(2p_i) \, p_i^2 \, \frac{\mu_i^2}{t \, e}.$$

*Further, we have*

$$(40) \qquad \|(U - \epsilon_0)^{j+1} \, \mathrm{CPo}(t, Q)\| \le 2^{j+1} \exp\left( - \min\left\{ t, \frac{j+1}{q_1} \right\} \right).$$

PROOF. Due to a simple coupling argument, we may assume that condition (C) is valid. By Lemma 5, it follows that, here,

$$\|(U - \epsilon_0)^{j+1} \, \mathrm{CPo}(t, Q)\| \le \left\| (U - \epsilon_0) \, \mathrm{CPo}\left( \frac{t}{j+1}, Q \right) \right\|^{j+1}$$

$$= 2^{j+1} \left( \sup_{m \in \mathbf{Z}_+} \mathrm{CPo}\left( \frac{t}{j+1}, Q \right)(\{m\}) \right)^{j+1}$$

$$\le \left( \frac{2(j+1)}{t \, e} \right)^{(j+1)/2},$$

where the latter bound is a consequence of

$$(41) \qquad \sup_{n \in \mathbf{Z}_+} \pi\left( n, \frac{t}{j+1} \right) \le \left( \frac{j+1}{2te} \right)^{1/2}, \qquad \sup_{m \in \mathbf{Z}_+} \sum_{n=0}^{\infty} Q^n(\{m\}) \le 1.$$

The second inequality in (41) follows from the fact that, if $T_1, T_2, \dots$ are independent and identically distributed random variables with $\mathcal{L}(T_1) = Q$, then, according to (C), we may assume that $T_i \ge 1$ ($i \in \mathbf{N}$) and therefore, for $m \in \mathbf{Z}_+$,

$$\sum_{n=0}^{\infty} Q^n(\{m\}) = \sum_{n=0}^{\infty} P\left( \sum_{i=1}^{n} T_i = m \right) = P\left( \bigcup_{n=0}^{\infty} \left\{ \sum_{i=1}^{n} T_i = m \right\} \right) \le 1.$$

Hence, (38) is shown. The bound (39) follows from (25), (35), and (38). In order to prove (40), it suffices to consider only the case $j = 0$ and $0 < tq_1 \leq 1$, since

$$\|(U - \epsilon_0)^{j+1} \, \mathrm{CPo}(t, \, Q)\| \leq \left\|(U - \epsilon_0) \, \mathrm{CPo}\Big( \min \Big\{ \frac{t}{j+1}, \, \frac{1}{q_1} \Big\}, \, Q \Big)\right\|^{j+1}.$$

Let $\varphi_1(z) = \exp(t(\sum_{r=1}^{\infty} q_r z^r - 1))$, and $\varphi_2(z) = (z - 1)\varphi_1(z)$, $(|z| \leq 1)$ denote the generating functions of $\mathrm{CPo}(t, \, Q)$ and $(U - \epsilon_0)\mathrm{CPo}(t, \, Q)$, respectively. In view of

$$\frac{d}{dz}\varphi_2(z) = \varphi_1(z)\Big( 1 - tq_1 + \sum_{r=1}^{\infty}(rq_r - (r+1)q_{r+1})z^r \Big),$$

we see that, since $tq_1 \leq 1$, the power series expansion of the derivative of $\varphi_2(z)$ has only non-negative coefficients. Therefore, in this case, we have $\varphi_2(z) = -e^{-t} + \sum_{m=1}^{\infty} B_m z^m$, where the $B_m$ are all non-negative. This yields

$$\|(U - \epsilon_0) \, \mathrm{CPo}(t, \, Q)\| = \varphi_2(1) + 2e^{-t} = 2e^{-t}$$

and completes the proof of (40). □

PROOF OF LEMMA 1. The assertions easily follow from (19), (28), (30), (35), (38), and an inequality in Roos (1999a, p. 132). □

### 4.3. *Proofs of the theorems*

PROOF OF THEOREM 2. Using (26), (32), and Stirling's formula, (14) is easily shown. To prove (15), we use (26), (32), and (33) and obtain

$$d_\tau \leq \min\{f_1(g(2)\,\beta_2(q_1^{-1})), \, 1\},$$

where

$$f_1(x) = \frac{x}{2^{3/2}} + x^2 + \frac{1}{2}\sum_{j=3}^{\infty} \frac{1}{j!}\Big( \frac{j\,x}{\sqrt{2}} \Big)^j, \qquad (x \geq 0).$$

If $x_1 \in (0, \, \infty)$ denotes the unique positive solution of $f_1(x) = 1$, then we obtain $d_\tau \leq g(2)\,\beta_2(q_1^{-1})/x_1$. Numerical computations yield $0.477 < x_1 < 0.478$ giving (15). The proofs of (16) and (17) are analogously shown by using (26), (32), (33), and (39). In particular, in the presence of (B) and (D), we obtain (17) in the following way:

$$d_\tau \leq \min\{f_2(g(2)\beta_2(1)), \, 1\} \leq \frac{g(2)}{x_2}\,\beta_2(1) \leq 9.2\,\beta_2(1),$$

where

$$f_2(x) = \frac{x}{e} + x^2 + \frac{1}{2}\sum_{j=3}^{\infty} \frac{1}{j!}\Big( \frac{2jx}{e} \Big)^j, \qquad (x \geq 0)$$

and $x_2 \in (0.459, 0.460)$ is the unique positive solution of $f_2(x) = 1$. This completes the proof of the theorem. $\square$

PROOF OF THEOREM 4.  In order to prove the second-order results, we use the equality

$$\mathcal{L}(S_n) - \mathrm{CPo}_2(\lambda, Q) = \frac{1}{3} \sum_{i=1}^{n} p_i^3 \, h(p_i(\epsilon_0 - Q_i)) \, (Q_i - \epsilon_0)^3 \, \mathrm{CPo}(\lambda, Q)$$

$$(42) \qquad\qquad + \sum_{j=2}^{n} \sum_{1 \le i(1) < \ldots < i(j) \le n} \prod_{s=1}^{j} \left( L_{i(s)} \, \mathrm{CPo}\left(\frac{\lambda}{j}, Q\right) \right),$$

which follows from (19) and (24). Using (42), we get

$$d_\tau' \le \frac{1}{6} \sum_{i=1}^{n} h(2p_i) \, p_i^3 \, \left\| (Q_i - \epsilon_0)^3 \, \mathrm{CPo}(\lambda, Q) \right\|$$

$$+ \frac{1}{2} \sum_{j=2}^{\infty} \frac{1}{j!} \left( \sum_{i=1}^{n} \left\| L_i \, \mathrm{CPo}\left(\frac{\lambda}{j}, Q\right) \right\| \right)^j.$$

Considering the summand for $j = 2$ separately, (20) is immediately shown by using (29), (31), (32), and (35). Inequality (21) is analogously shown by using (35), (31), (32), (38), (39), and (42). Theorem 4 is proved. $\square$

Theorem 5 can easily be proved by using (19), (30), (35), (36), and (40).

PROOF OF THEOREM 1.  First, let us mention that, for arbitrary $k \in \mathbf{N}$,

$$d_\tau \le d_{\mathrm{TV}}(S_n, S_n^{(k)}) + d_{\mathrm{TV}}(\mathcal{L}(S_n^{(k)}), \mathrm{CPo}(\lambda^{(k)}, Q^{(k)}))$$

$$+ d_{\mathrm{TV}}(\mathrm{CPo}(\lambda^{(k)}, Q^{(k)}), \mathrm{CPo}(\lambda, Q)),$$

where $\lambda^{(k)}$ and $Q^{(k)}$ are defined as in (34) (with $t = \lambda$) and $S_n^{(k)}$ is the sum of independent random variables $X_1^{(k)}, \ldots, X_n^{(k)}$ with distributions

$$\mathcal{L}(X_i^{(k)}) = \epsilon_0 + \sum_{r=1}^{k} p_i q_{i,r}(U_r - \epsilon_0), \qquad (i \in \{1, \ldots, n\}).$$

It is not difficult to show that

$$d_{\mathrm{TV}}(S_n, S_n^{(k)}) \overset{(k \to \infty)}{\longrightarrow} 0, \qquad d_{\mathrm{TV}}(\mathrm{CPo}(\lambda^{(k)}, Q^{(k)}), \mathrm{CPo}(\lambda, Q)) \overset{(k \to \infty)}{\longrightarrow} 0.$$

Further, from Roos (1999a, Theorem 1) and a simple coupling argument, it follows that

$$\lim_{k \to \infty} d_{\mathrm{TV}}(\mathcal{L}(S_n^{(k)}), \mathrm{CPo}(\lambda^{(k)}, Q^{(k)})) \le \frac{\alpha_1(2^{-3/2})}{1 - 2\, e\, \alpha_1(2^{-3/2})},$$

$$\lim_{k \to \infty} d_{\mathrm{TV}}(\mathcal{L}(S_n^{(k)}), \mathrm{CPo}(\lambda^{(k)}, Q^{(k)})) \le 8.8\, \beta_1,$$

where, for the first inequality, we assume that $\alpha_1(2^{-3/2}) < 1/(2e)$. This leads to (12) and (13). $\qquad\square$

The proof of Theorem 3 is similar to the preceeding one and is therefore omitted. Note that here we used Theorem 2 of Roos (1999a).

**Acknowledgment.** The author is grateful to V. Čekanavičius for his valuable comments, which led to an improvement of the paper.

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FACHBEREICH MATHEMATIK, SPST
UNIVERSITÄT HAMBURG
BUNDESSTR. 55
20146 HAMBURG
GERMANY
E-MAIL: roos@math.uni-hamburg.de