Mixing Taylor shifts and universal Taylor series

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October 15, 2014

Abstract

It is known that, generically, Taylor series of functions holomorphic in a simply connected complex domain exhibit maximal erratic behaviour outside the domain (so-called universality) and overconvergence in parts of the domain. Our aim is to show how the theory of universal Taylor series can be put into the framework of linear dynamics. This leads to a unified approach to universality and overconvergence and yields new insight into the boundary behaviour of Taylor series.

Key words: backward shift, mixing operator, universality
2010 Mathematics subject classification: 30B30, 30K05, 47A16.

1 Mixing Taylor shifts

By $\mathbb{C}_\infty$ we denote the extended complex plane endowed with the spherical metric. For $\Omega \subset \mathbb{C}_\infty$ open, $H(\Omega)$ is the space of holomorphic functions on Ω that vanish at infinity in case that $\infty \in \Omega$. This space is endowed with the usual topology of uniform convergence on compact sets. In case $0 \in \Omega$ we define $T = T_\Omega : H(\Omega) \to H(\Omega)$ by

$$Tf(z) := \frac{f(z) - f(0)}{z} \quad (z \neq 0), \qquad Tf(0) := f'(0)$$

(with $w/\infty := 0$ for $w \in \mathbb{C}$). If $f(z) = \sum_{\nu=0}^{\infty} a_\nu z^\nu$ then

$$Tf(z) = \sum_{\nu=0}^{\infty} a_{\nu+1} z^\nu$$

for $|z|$ sufficiently small. We call T the Taylor (backward) shift on $H(\Omega)$. Backward shifts are studied intensively on Hardy spaces on the unit disc $\mathbb{D}$ (see e.g. [7], [17]).

By induction it is easily seen that

$$T^n f(z) = \frac{f - s_{n-1} f(z)}{z^n} \quad (z \neq 0), \qquad T^n f(0) = a_n,$$

for $n \in \mathbb{N}_0$, where $s_n f(z) := \sum_{\nu=0}^{n} a_\nu z^\nu$ denotes the n-th partial sum of the Taylor expansion of f about 0 (and $s_{-1} f := 0$).

It is readily seen that the Cauchy kernel provides a family of eigenfunctions for T. More precisely, for $\alpha \in \mathbb{C}$ and $M \subset \mathbb{C}_\infty \setminus \{\alpha\}$ we define

$$\gamma_{\alpha,M}(z) = 1/(\alpha - z) \quad (z \in M).$$

If we put $\gamma_{\infty,M} := 1$ for $M \subset \mathbb{C}$ and write

$$K := K(\Omega) := \mathbb{C}_\infty \setminus \Omega$$

then, for each $\alpha \in K$, the function $\gamma_\alpha = \gamma_{\alpha,\Omega}$ is an eigenfunction corresponding to the eigenvalue $1/\alpha$. In particular, the set $K^{-1} := \{1/\alpha : \alpha \in K\}$ is contained in the point spectrum of T. On the other hand, one observes that, in case $\alpha \in \Omega \setminus \{0\}$,

$$S_\alpha g(z) := \begin{cases} \dfrac{zg(z) - \alpha g(\alpha)}{1 - z/\alpha} & (\alpha \neq \infty), \\ zg(z) - g'(\infty) & (\alpha = \infty) \end{cases}$$

(continuously extended at the point α) defines the continuous inverse operator to $T - \alpha^{-1} I$ (with I being the identity operator on $H(\Omega)$). This shows that the spectrum of T on $H(\Omega)$ equals K^{-1}. In particular, T is invertible if and only if $\infty \in \Omega$ and, in this case,

$$T^{-1} g(z) = \sum_{\nu=0}^{\infty} \frac{b_{\nu+1}}{z^{\nu+1}}$$

near ∞ if $g(z) = \sum_{\nu=0}^{\infty} b_\nu / z^{\nu+1}$ near ∞, hence T^{-1} is again a backward shift.

For notions from topological dynamics and linear dynamics used in the sequel we refer to [1] and [11]. In particular, T is hypercyclic on $H(\Omega)$ if there is a function $f \in H(\Omega)$ having dense orbit $\{T^n f : n \in \mathbb{N}\}$ in $H(\Omega)$, and T is mixing on $H(\Omega)$ if for each pair of non-empty open sets U, V in $H(\Omega)$ a positive integer N exist with $T^n(U) \cap V \neq \emptyset$ for all $n \geq N$. We finally say that a property is satisfied for comeagre many elements of a complete metric space, if the property is satisfied on a residual set in the space.

Theorem 1.1. *Let $\Omega \subset \mathbb{C}_\infty$ be open with $0 \in \Omega$. Then the following are equivalent:*

a) T is mixing on $H(\Omega)$,

b) T is hypercyclic on $H(\Omega)$,

c) each connected component of K intersects the unit circle $\mathbb{T}$.

Proof. We show that c) implies a): Let $0 < \delta < \operatorname{dist}(0, \partial\Omega)$. For $m \in \mathbb{N}$ we set

$$K_m := \{z \in \mathbb{C}_\infty : \operatorname{dist}(z, K) \leq \delta/m\}$$

and $\Omega_m := \mathbb{C}_\infty \setminus K_m$. Then (Ω_m, Ω) is a Runge pair and thus

$$H(\Omega) = \operatorname{proj}(H(\Omega_m))_{m \in \mathbb{N}}$$

is strongly reduced (for a definition, see [11, p. 340]).

We show that $T_m := T_{\Omega_m}$ is mixing on $H(\Omega_m)$ for all $m \in \mathbb{N}$. This yields that T is mixing on $H(\Omega)$ due to the dynamical transference principle stated in [11, Proposition 12.18].

By a variant of Runge's theorem, $\mathrm{lin}\{\gamma_\alpha : \alpha \in A\}$ is dense in $H(\Omega_m)$ if $A \subset K_m$ has an accumulation point in every connected component of K_m. Since each connected component of K meets $\mathbb{T}$, the same is true for each connected component of the interior of K_m. Thus, $\mathrm{lin}\{\gamma_\alpha : \alpha \in K_m, |\alpha| > 1\}$ and $\mathrm{lin}\{\gamma_\alpha : \alpha \in K_m, |\alpha| < 1\}$ are dense in $H(\Omega_m)$. The Godefroy-Shapiro criterion [11, Theorem 3.1] yields that T_m is mixing.

The implication a) to b) is an immediate consequence of the Birkhoff transitivity theorem ([11, Theorem 2.19]).

For the proof of the implication b) to c) we assume that there is a connected component L_0 of K that does not intersect the unit circle. Then L_0 is contained in the interior of $\mathbb{T}$ or in the exterior. Therefore, one can find a clopen subset L of K with $L_0 \subset L$ and so that L is contained in the open unit disc or in the complement of the closed unit disc (see e.g. [1, Lemma 1.21]). It follows that every $f \in H(\Omega)$ can be written as $f = f_0 + f_1$ with $f_0 \in H(\mathbb{C}_\infty \setminus L)$, $f_1 \in H(\mathbb{C}_\infty \setminus (K \setminus L))$, and by the inverse mapping theorem this defines an isomorphism between the Fréchet spaces $H(\Omega)$ and $H(\mathbb{C}_\infty \setminus L) \oplus H(\mathbb{C}_\infty \setminus (K \setminus L))$. Thus, if T is hypercyclic or, equivalently, topologically transitive on $H(\Omega)$ then so is T on $H(\mathbb{C}_\infty \setminus L)$ (see [11, Proposition 1.42]). We write $U := \mathbb{C}_\infty \setminus L$.

As a first case we assume that $L \subset \{z : |z| > 1\}$, that is, $\overline{\mathbb{D}} \subset U$. If $f_0 \in H(U)$ then $(f_0 - s_{n-1} f_0)$ tends to 0 uniformly on a circle $\{z : |z| = R\} \subset U$, for some $R > 1$, and therefore also $(T^n f_0)$. The maximum principle shows that the convergence is uniform on $\{|z| \leq R\}$. This contradicts the hypercyclicity of T_U.

We assume now that $L \subset \mathbb{D}$. Since $\infty \in U$ in that case, the operator T_U is invertible and a similar argument as above applies to T_U^{-1}. Indeed: If $g \in H(U)$, then the partial sums $(t_{n-1} g)$ of the expansion $g(z) = \sum_{\nu=0}^\infty b_\nu / z^{\nu+1}$ tend to g uniformly on $\{|z| = r\}$, for some $r < 1$. The maximum principle implies that

$$(T_U^{-1})^n g(z) = z^n \Big(g(z) - (t_{n-1} g)(z) \Big)$$

tends to 0 locally uniformly on $\{|z| \geq r\}$. Then T_U^{-1} and, equivalently, T_U cannot be hypercyclic. □

Remark 1.2. 1. Let Ω be so that T is mixing on $H(\Omega)$, that is, each connected component of K meets $\mathbb{T}$. If $\Lambda \subset \mathbb{N}$ is infinite, then $(T^n)_{n \in \Lambda}$ is topologically transitive on $H(\Omega)$ (that is, $(T^{n_k})_{k \in \mathbb{N}}$, where $\Lambda = \{n_k : k \in \mathbb{N}\}$ with $n_{k+1} > n_k$, is topologically transitive). The Universality Criterion ([11, Theorem 1.57]) shows that comeagre many $f \in H(\Omega)$ are universal for $(T^n)_{n \in \Lambda}$, i.e. for comeagre many $f \in H(\Omega)$, we have that $\{T^n f : n \in \Lambda\}$ is dense in $H(\Omega)$.

2. Let Ω be a domain. In connection with the chaoticity of T it is interesting to have information about periodic points of T. It is easily seen that a function $f \in H(\Omega)$ is periodic for T if and only if K contains roots of unity and f is of

the form

$$f(z) = \frac{p(z)}{1 - z^k} \qquad (z \in \Omega)$$

for some polynomial p of degree $< k$ and some k (and for poles belonging to K). This implies that T is chaotic on $H(\Omega)$ if and only if each connected component of K contains infinitely many roots of unity. Moreover, the classical Kronecker theorem concerning the characterization of Taylor series of rational functions shows that (exactly) for rational functions f the span of $\{T^n f : n \in \mathbb{N}_0\}$ is finite dimensional. Thus, rational f are in a sense far from being cyclic (and therefore also far from being hypercyclic).

3. In the special case $\Omega = \mathbb{C}_\infty \setminus \{1/\alpha\}$, the equivalence of b) and c) in Theorem 1.1 is already proved in [2, Theorem 3.30] in a different way. To be more precise, let Exp_α denote the (Fréchet) space of entire functions of exponential type having indicator diagram $\{\alpha\}$, that is, functions of the form $F(w) = e^{\alpha w} F_0(w)$ for some F_0 of subexponential type (see, e.g. [5]). Then T_Ω is conjugate to the differentiation operator $DF := F'$ on Exp_α (essentially via the Borel transform) and D is hypercyclic if and only if $|\alpha| = 1$ ([2, Theorem 3.26] or [3, Theorem 2.4]).

4. It is clear that, for $0 < \lambda < \infty$, Theorem 1.1 holds for the operator λT when $\mathbb{T}$ is replaced by $\lambda^{-1}\mathbb{T}$. That is, λT is mixing (or hypercyclic) on $H(\Omega)$ if and only if each connected component of K intersects $\lambda^{-1}\mathbb{T}$. Therefore, all the results given in the following section also hold for $\mathbb{D}$ and $\mathbb{T}$ being replaced by $\lambda^{-1}\mathbb{D}$ and $\lambda^{-1}\mathbb{T}$, respectively.

2 Applications

We focus now on the partial sums of the Taylor series $\sum_{\nu=0}^{\infty} a_\nu z^\nu$ of $f \in H(\Omega)$. We have

$$|T^{n+1} f| \geq |f - s_n f|$$

on $\overline{\mathbb{D}} \cap \Omega$ and

$$|T^{n+1} f| \leq |f - s_n f|$$

on $(\mathbb{C} \setminus \mathbb{D}) \cap \Omega$. Theorem 1.1 immediately implies

Corollary 2.1. *Let Ω be so that each component of K meets the unit circle and let $\Lambda \subset \mathbb{N}_0$ be infinite. Then for comeagre many f in $H(\Omega)$ there is a subsequence of $(s_n f)_{n\in\Lambda}$ tending to f locally uniformly on $\overline{\mathbb{D}} \cap \Omega$ and a subsequence of $(s_n f)_{n\in\Lambda}$ tending to ∞ locally uniformly on $(\mathbb{C} \setminus \mathbb{D}) \cap \Omega$.*

Proof. Let, according to Remark 1.2, f be a universal function for $(T^{n+1})_{n\in\Lambda}$. Then there is a sequence (n_j) in Λ with $T^{n_j+1} f \to 0$ in $H(\Omega)$ as $j \to \infty$ and, hence,

$$s_{n_j} f \to f \qquad (j \to \infty) \tag{1}$$

locally uniformly on $\overline{\mathbb{D}} \cap \Omega$.

Similarly, there is a sequence (m_j) in Λ with $T^{m_j+1}f \to \infty$ locally uniformly on $\mathbb{C} \cap \Omega$ as $j \to \infty$ and thus

$$s_{m_j} f \to \infty \qquad (j \to \infty)$$

locally uniformly on $(\mathbb{C} \setminus \mathbb{D}) \cap \Omega$. □

If $K \cap \mathbb{D} = \emptyset$ then the corollary (mainly) gives a statement on the behaviour of the partial sums $s_n f$ on $\mathbb{T}$. In particular, for comeagre many $f \in H(\Omega)$, a subsequence of $(s_n f)$ tends to f locally uniformly on $\mathbb{T} \cap \Omega$. In the special case of the punctured plane $\Omega = \mathbb{C}_\infty \setminus \{1\}$ it is known that finite limit functions outside of $\overline{\mathbb{D}}$ can only exist on polar sets (see [4], [13]). So the generic behaviour of the partial sums on $\mathbb{T}$ is in clear contrast to the behaviour outside of $\overline{\mathbb{D}}$.

An interesting question is whether there are (finite) limit functions different from f on parts of $\mathbb{T} \cap \Omega$. A recent result of Gardiner and Manolaki (see [10]) which is based on deep tools from potential theory shows that functions $f \in H(\mathbb{D})$ have the following remarkable property:

Let $(s_{n_k} f)$ be an arbitrary subsequence of $(s_n f)$ converging to a (finite) limit function h pointwise on a subset A of $\mathbb{T}$. If f has nontangential limits $f^*(\zeta)$ for $\zeta \in A$, then $h = f^*$ almost everywhere (with respect to arc length measure) on A.

In particular, the theorem proves the special attraction of the "right" boundary function as a limit function in the case that f extends continuously to some subarc of $\mathbb{T}$. On the other hand, the following result implies that, on small subsets of $\mathbb{T}$, even for functions that are holomorphically extendable a maximal set of uniform limit functions may exist.

We recall that a closed subset of $\mathbb{T}$ is called a Dirichlet set if a subsequence of (z^n) tends to 1 uniformly on E.

Theorem 2.2. *Let Ω be so that each component of K meets the unit circle and let $E \subset \mathbb{T} \cap \Omega$ be a Dirichlet set. Then comeagre many $f \in H(\Omega)$ enjoy the property that, for each $h \in C(E)$, a subsequence of $(s_n f)$ tends to h uniformly on E.*

Proof. Let $\Lambda \subset \mathbb{N}_0$ be infinite with $z^{n+1} \to 1$ uniformly on E as $n \to \infty$, $n \in \Lambda$. If we fix some $\alpha \in K$, according to a variant of Mergelian's theorem (note that E has connected complement), $\{g|_E : g \in H(\mathbb{C}_\infty \setminus \{\alpha\})\}$ is dense in $C(E)$, endowed with the uniform norm. So we can assume $h \in H(\Omega)$.

Let f be universal for $(T^{n+1})_{n \in \Lambda}$ (which, according to Remark 1.2, is the case for comeagre many f in $H(\Omega)$). Then there are n_j in Λ with $T^{n_j+1}f \to f - h$ $(j \to \infty)$ locally uniformly on Ω and thus in particular uniformly on E. Then also

$$z^{n_j+1}T^{n_j+1}f(z) \to (f-h)(z) \qquad (j \to \infty)$$

uniformly on E and therefore

$$s_{n_j} f(z) = f(z) - z^{n_j+1}T^{n_j+1}f(z) \to h(z) \qquad (j \to \infty)$$

uniformly on E. □

Remark 2.3. 1. It is well-known that each finite set in $\mathbb{T}$ is a Dirichlet set and that there are countable sets which are not Dirichlet. Moreover, Dirichlet sets cannot have positive arc length measure (as also follows from the above results), but can have Hausdorff dimension 1 (see e.g. [12]). As remarked above, in the special case $\Omega = \mathbb{C}_\infty \setminus \{1\}$ finite limit functions outside of $\overline{\mathbb{D}}$ can only exist on polar sets (and thus on sets of vanishing Hausdorff dimension). Theorem 2.2 shows that the situation is essentially different on $\mathbb{T}$.

2. In [6], the concept of so-called pseudo Dirichlet sets (or D-sets) was introduced. Exhausting such sets by an increasing sequence of Dirichlet sets, it is possible to extend the assertion of Theorem 2.2 to appropriate pointwise limit functions on pseudo Dirichlet sets.

We focus now on the case that Ω has at least two components. We write Ω_0 for the component of Ω containing the origin and set $U := \Omega \setminus \Omega_0$.

Theorem 2.4. *Let Ω be so that each component of K meets the unit circle and let $\Lambda \subset \mathbb{N}_0$ be infinite. Then comeagre many $f_0 \in H(\Omega_0)$ enjoy the following property: For each $g \in H(U)$ a sequence (n_j) in Λ exists with $s_{n_j} f_0 \to f_0$ locally uniformly on $\overline{\mathbb{D}} \cap \Omega_0$ and $s_{n_j} f_0 \to g$ locally uniformly on $\overline{\mathbb{D}} \cap U$.*

Proof. If U is empty, the result follows immediately from Theorem 1.1 and (1). Let U be nonempty. Then $H(\Omega)$ is in an obvious way isomorphic to $H(\Omega_0) \oplus H(U)$ (for the corresponding notions, see e.g. [11, p. 37]). We write A for the set of all $f = (f_0, g) \in H(\Omega_0) \oplus H(U)$ having the property that there is a sequence (n_j) in Λ with $s_{n_j} f_0 = s_{n_j} f \to f$ locally uniformly on $\overline{\mathbb{D}} \cap \Omega$. By definition, this is the same as saying that $(s_{n_j} f_0)$ tends to f_0 locally uniformly on $\overline{\mathbb{D}} \cap \Omega_0$ and to g locally uniformly on $\overline{\mathbb{D}} \cap U$. According to Corollary 2.1, the set A is residual in $H(\Omega_0) \oplus H(U)$. Thus, the Kuratowski-Ulam theorem (see, for example, [14, p. 53]) implies that, for comeagre many $f_0 \in H(\Omega_0)$, the set A_{f_0} of $g \in H(U)$ such that $s_{n_j} f_0 \to g$ locally uniformly on $\overline{\mathbb{D}} \cap U$ and $s_{n_j} f_0 \to f_0$ locally uniformly on $\overline{\mathbb{D}} \cap \Omega_0$ for some sequence (n_j) in Λ, is residual in $H(U)$. Finally, a diagonal argument shows that A_{f_0} is closed in $H(U)$. So we have $A_{f_0} = H(U)$ for comeagre many $f \in H(\Omega_0)$. □

Example 2.5. (Universality on open sets; cf. [15], [16])
Let $\Omega \subset \mathbb{D}$ have simply connected components. Then comeagre many $f_0 \in H(\Omega_0)$ enjoy the following property: Given $g \in H(U)$, there exists a subsequence of $(s_n f_0)$ tending to f_0 locally uniformly on Ω_0 and to g locally uniformly on U.

Finally we apply the above concept to the case of uniform universality on compact sets (cf. [16], [8], [18]):

For $E \Subset \mathbb{C}$ we consider the closed subspace $A(E)$ of $C(E)$ of functions being holomorphic in the interior of E (again, $C(E)$ is endowed with the uniform norm). If $0 \notin E$ and if $\Omega_0 \subset \mathbb{C}_\infty \setminus E$ is a domain with $0 \in \Omega_0$, we may consider $T = T_{\Omega_0,E}$ also as a mapping on $H(\Omega_0) \oplus A(E)$, defined by

$$T_{\Omega_0,E}(f_0, g) := (T_{\Omega_0} f_0, E \ni z \mapsto (g(z) - f_0(0))/z).$$

If $\mathbb{C}\setminus(\Omega_0 \cup E)$ is non-empty then, for $\alpha \in \mathbb{C}\setminus(\Omega_0 \cup E)$, the functions $(\gamma_{\alpha,\Omega_0}, \gamma_{\alpha,E})$ again form a family of eigenfunctions to the eigenvalues $1/\alpha$.

We suppose now, in addition, that E has connected complement and that each component of $\partial\Omega_0$ meets $\mathbb{T}$. A modification of the proof of Theorem 1.1 implies that $T = T_{\Omega_0,E}$ is again mixing.

(Indeed: Defining Ω_m as in the proof of Theorem 1.1 with Ω_0 instead of Ω, we have

$$H(\Omega_0) \oplus A(E) = \text{proj}(H(\Omega_m) \oplus A(E))_{m\in\mathbb{N}}.$$

Applying the dynamical transference principle, it suffices to show that $T_{\Omega_m,E}$ is mixing on $H(\Omega_m) \oplus A(E)$. This follows again by the Godefroy-Shapiro criterion and the fact that, according to Mergelian's theorem, rational functions with a fixed pole in one point outside E are dense in $A(E)$.)

A remarkable fact is that, in the maximal (and most interesting) case of $\Omega_0 = \mathbb{C}_\infty \setminus E$, no eigenfunctions exist. The change to Ω_m, however, supplies us with a sufficiently large family of eigenfunctions.

Now we can argue in the same way as above using the Kuratowski-Ulam theorem to obtain the following result

Theorem 2.6. *Let $E \Subset \mathbb{C}$ have connected complement. Moreover, let $\Omega_0 \subset \mathbb{C}_\infty \setminus E$ be a domain and suppose that each component of $\partial\Omega_0$ meets the unit circle. Then, for each infinite set $\Lambda \subset \mathbb{N}_0$, comeagre many $f_0 \in H(\Omega_0)$ enjoy the following property: For each $g \in A(E)$ a sequence (n_j) in Λ exists with $s_{n_j} f_0 \to f_0$ locally uniformly on $\overline{\mathbb{D}} \cap \Omega_0$ and $s_{n_j} f_0 \to g$ uniformly on $\overline{\mathbb{D}} \cap E$.*

Remark 2.7. We consider the case $\Omega_0 = \mathbb{C}_\infty \setminus E$. For E being the closed disc with centre x and radius $1 - x$, where $1/2 < x < 1$, we get Theorem 4.4 from [8] (with some extra information on the behaviour on the unit circle). Moreover, for general E we obtain the main result from [18], again with extra information on the overconvergence to f_0 on the set $\overline{\mathbb{D}} \cap \Omega_0$. Using advanced tools from potential theory, Gardiner has proved in [9, p. 248] that boundedness of a subsequence of $(s_n f_0)$ on discs E already *implies* overconvergence of the same subsequence in a nontrivial part of Ω_0.

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