

(1)

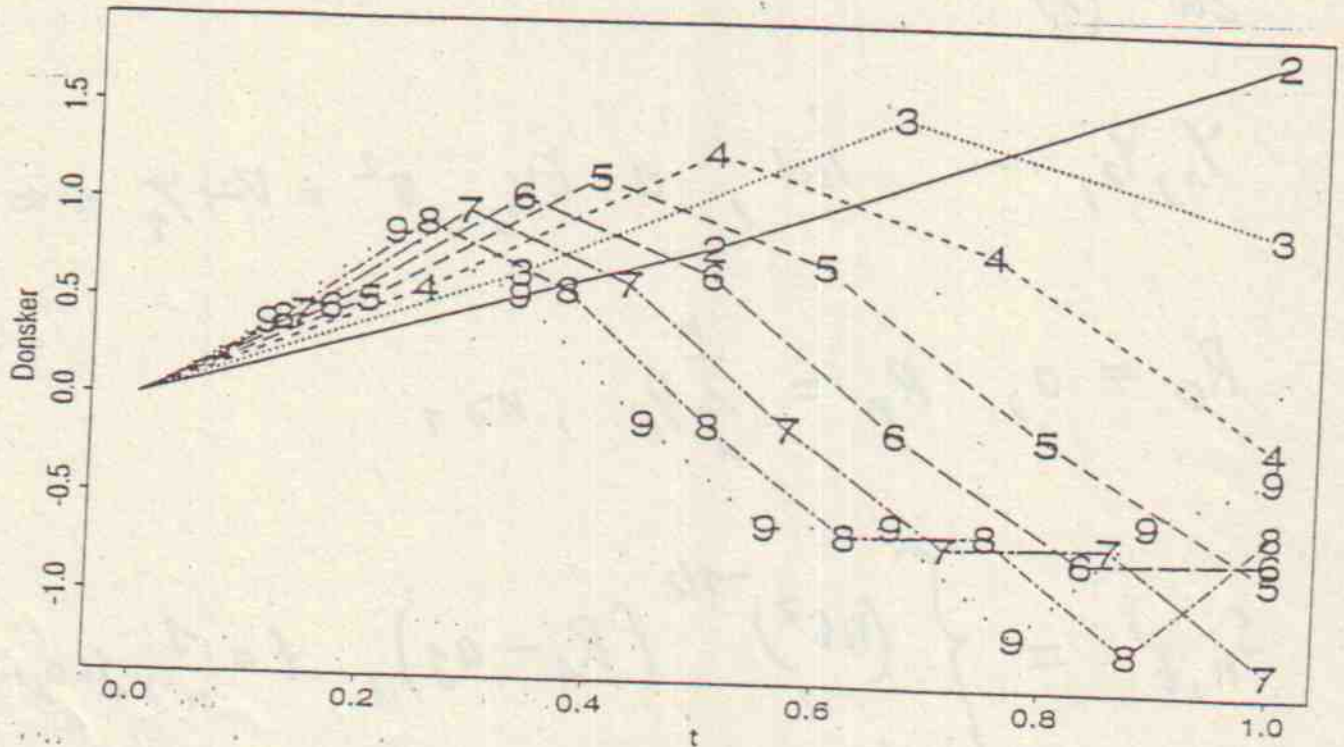


Figure 1.3.11 Sample paths of the process  $S_n$  for one sequence of realizations  $Y_1(\omega), \dots, Y_9(\omega)$  and  $n = 2, \dots, 9$ .

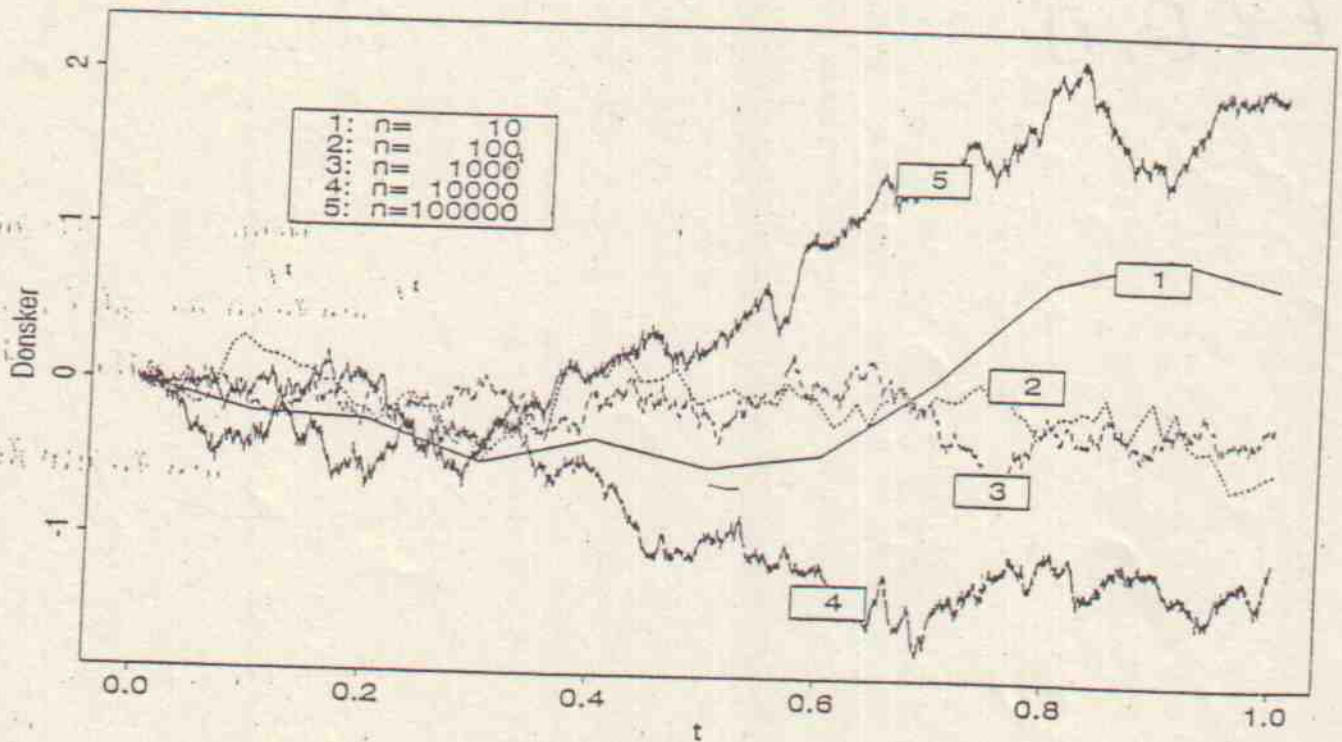


Figure 1.3.12 Sample paths of the process  $S_n$  for different  $n$  and the same sequence of realizations  $Y_1(\omega), \dots, Y_{100,000}(\omega)$ .

zu (1)

$Y_1, Y_2, \dots$  iid,  $a = EY_1$ ,  $\sigma^2 = \text{Var} Y_1 < \infty$

$$R_0 = 0, \quad R_n = \sum_{i=1}^n Y_i, \quad n \geq 1$$

$$S_{n,t} = \begin{cases} (n\sigma^2)^{-1/2} (R_j - a_j), & t = \frac{j}{n}, j=0, \dots, n \\ \text{linear interpoliert, sonst} \end{cases}$$

$$t \in [0, 1]$$

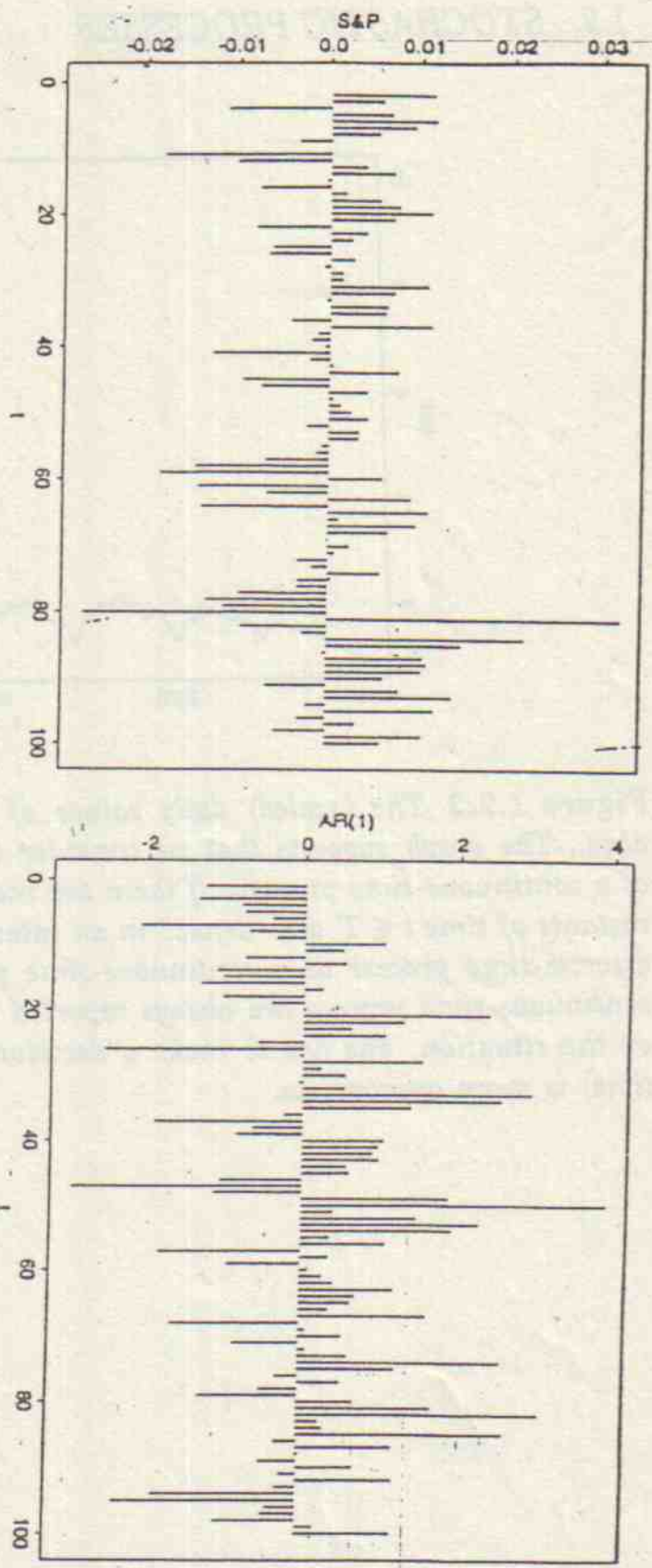


Figure 1.2.4 Two time series  $X_t, t = 1, \dots, 100$ . Left: 100 successive daily log-returns of the S&P index; see Figure 1.1.4. Right: a simulated sample path of the autoregressive process  $X_t = 0.5X_{t-1} + Z_t$ , where  $Z_t$  are iid  $N(0,1)$  random variables; see Example 1.2.3.



## 1.2. STOCHASTIC PROCESSES

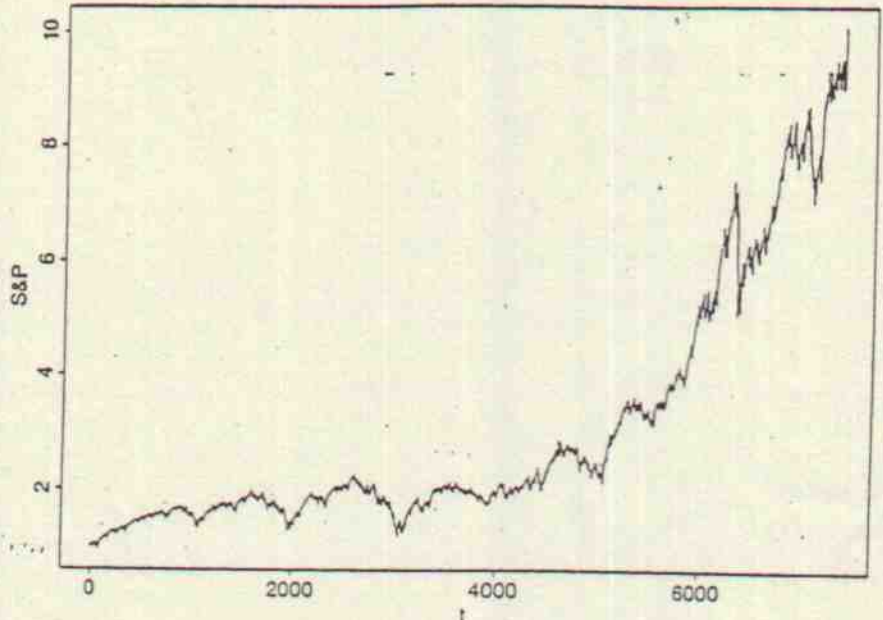


Figure 1.2.2 The (scaled) daily values of the S&P index over a period of 7,422 days. The graph suggests that we consider the S&P time series as the sample path of a continuous-time process. If there are many values in a time series such that the instants of time  $t \in T$  are "dense" in an interval, then one may want to interpret this discrete-time process as a continuous-time process. The sample paths of a real-life continuous-time process are always reported at discrete instants of time. Depending on the situation, one has to make a decision which model (discrete- or continuous-time) is more appropriate.