16. Some Symmetric Distributions

1. Standard Normal Distribution: $N(0,1)$

(λ-Dichte) $$f(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2}), \qquad -\infty < x < \infty.$$

2. Logistic Distribution $L(1)$

$$f(x) = \frac{\exp(-x)}{\{1+\exp(-x)\}^2}, \qquad -\infty < x < \infty.$$

3. (~~Laplace Distribution~~) Doppelexponential-Vert.

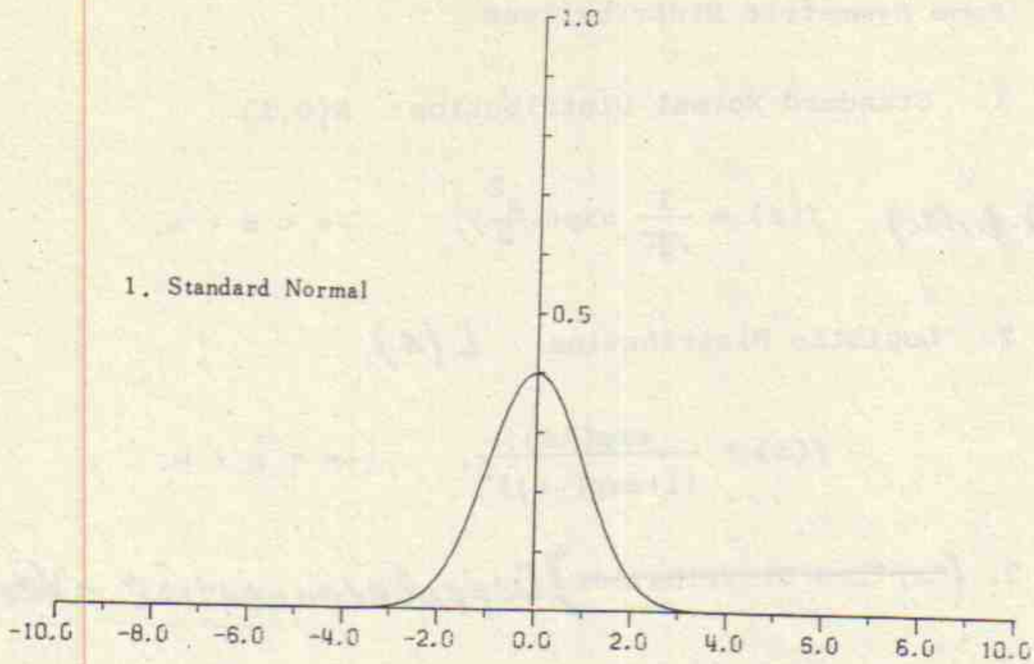
$$f(x) = \frac{1}{2} \exp(-|x|), \qquad -\infty < x < \infty.$$

4. t Distribution with 5 Degrees of Freedom: $t(5) = t_5$

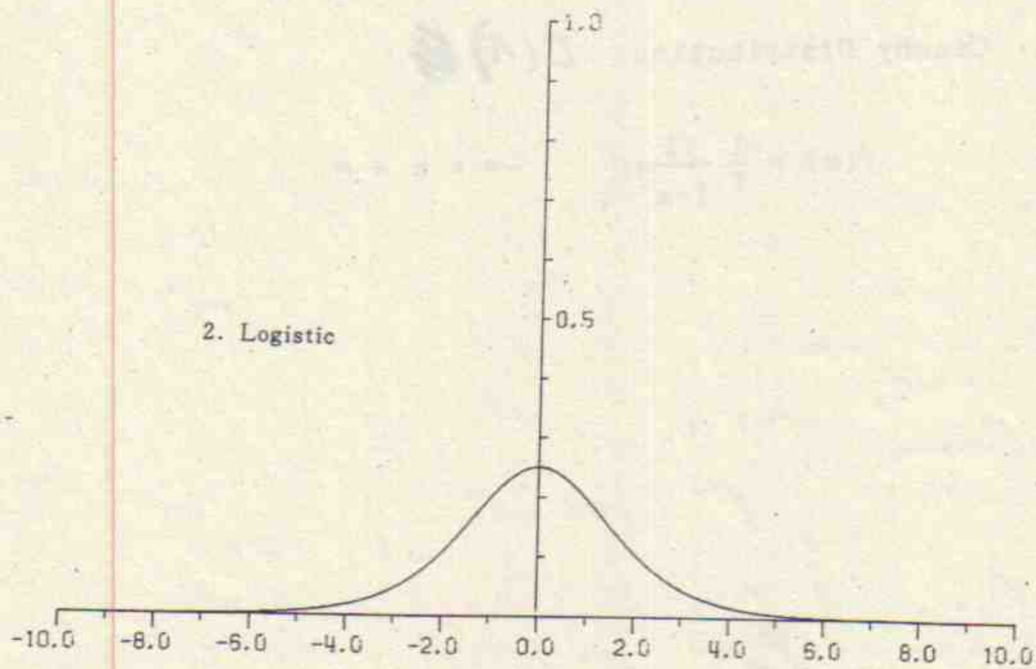
$$f(x) = \frac{\Gamma(3)}{\Gamma(\frac{5}{2})\sqrt{5\pi}} (1+\frac{x^2}{5})^{-3}, \qquad -\infty < x < \infty.$$

5. Cauchy Distribution $C(1)$

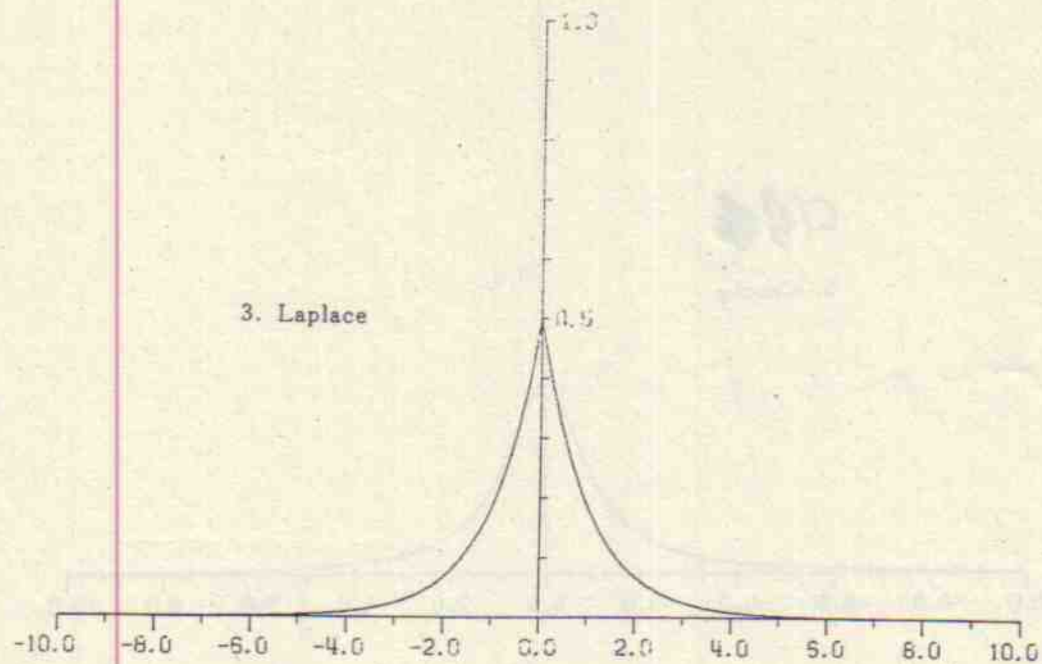
$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}, \qquad -\infty < x < \infty.$$

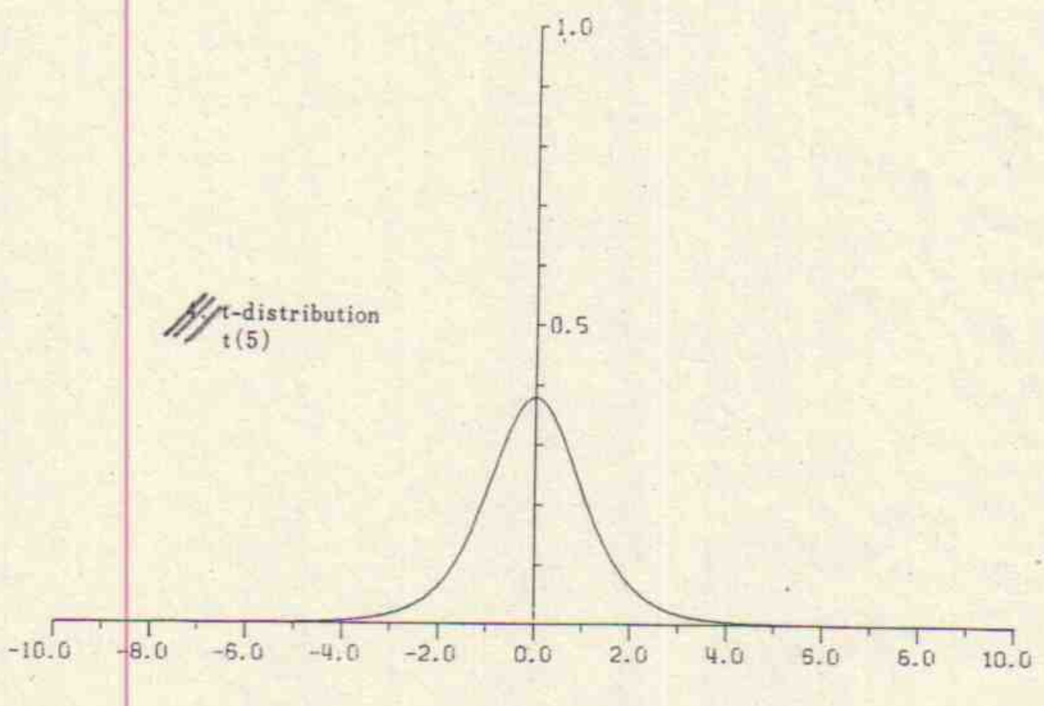


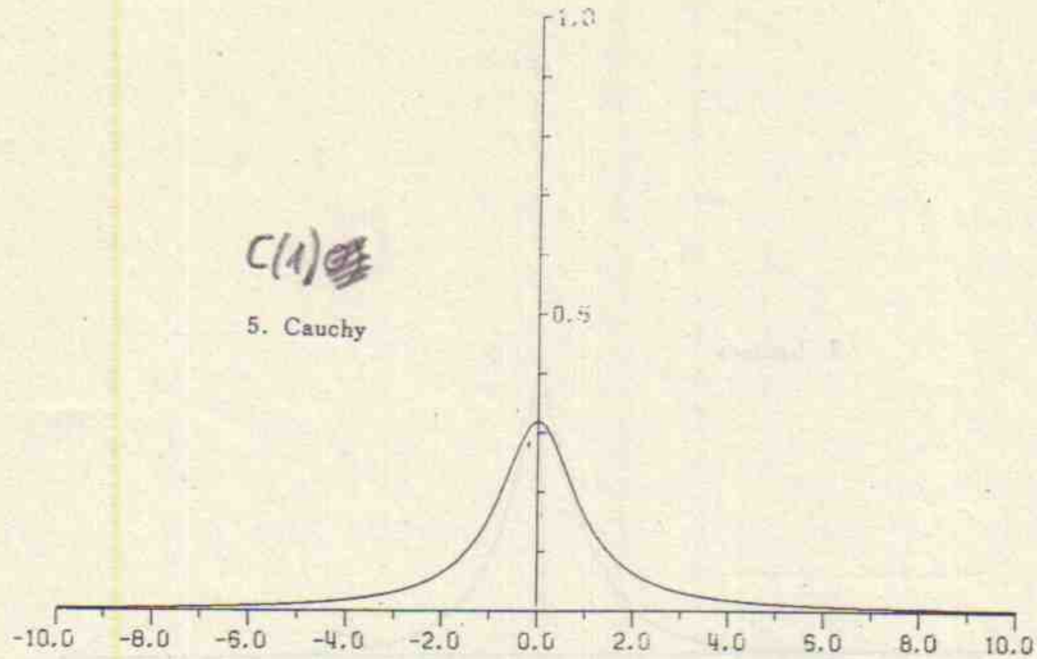
1. Standard Normal



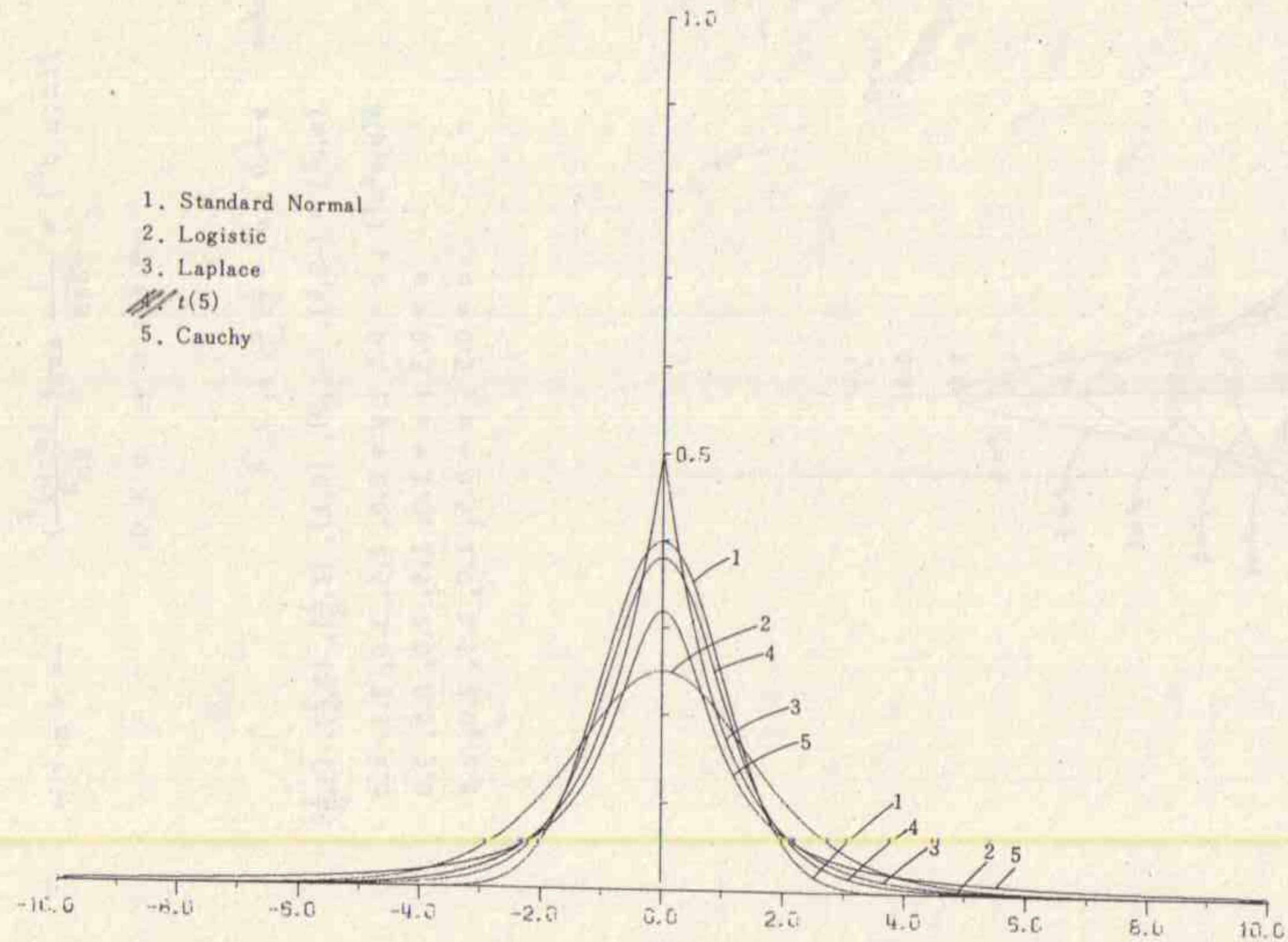
2. Logistic


1.0
3. Laplace
0.5
-10.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0
10.0


1.0
t-distribution
t(5)
0.5
-10.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0
10.0



5. Cauchy


1. Standard Normal
2. Logistic
3. Laplace
4. t(5)
5. Cauchy
1.0
0.5
-10.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0
10.0
1
2
4
3
5
1
4
3
2
5

8. Normal(Gaussian) Distribution: $N(\mu,\sigma^2)$ $= N(a,\sigma^2),\ a \hat{=} \mu$

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}, \qquad -\infty < x < \infty,$$

$$-\infty < \mu < \infty, \quad \sigma > 0.$$

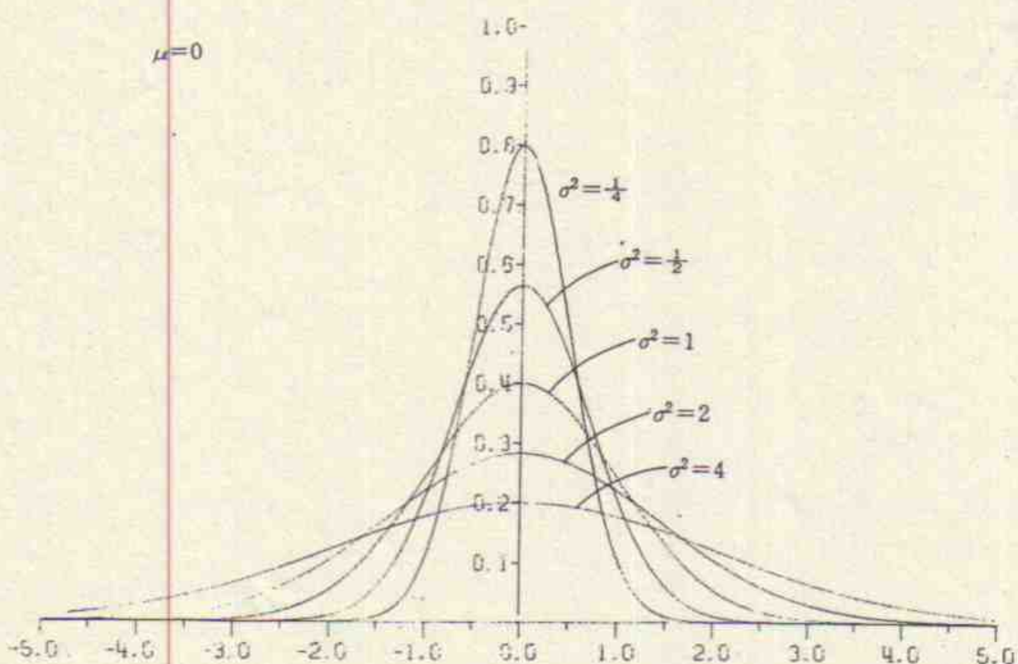
Figures $\mu = 0$; $\sigma^2 = \frac{1}{4}, \frac{1}{2}, 1, 2, 4$

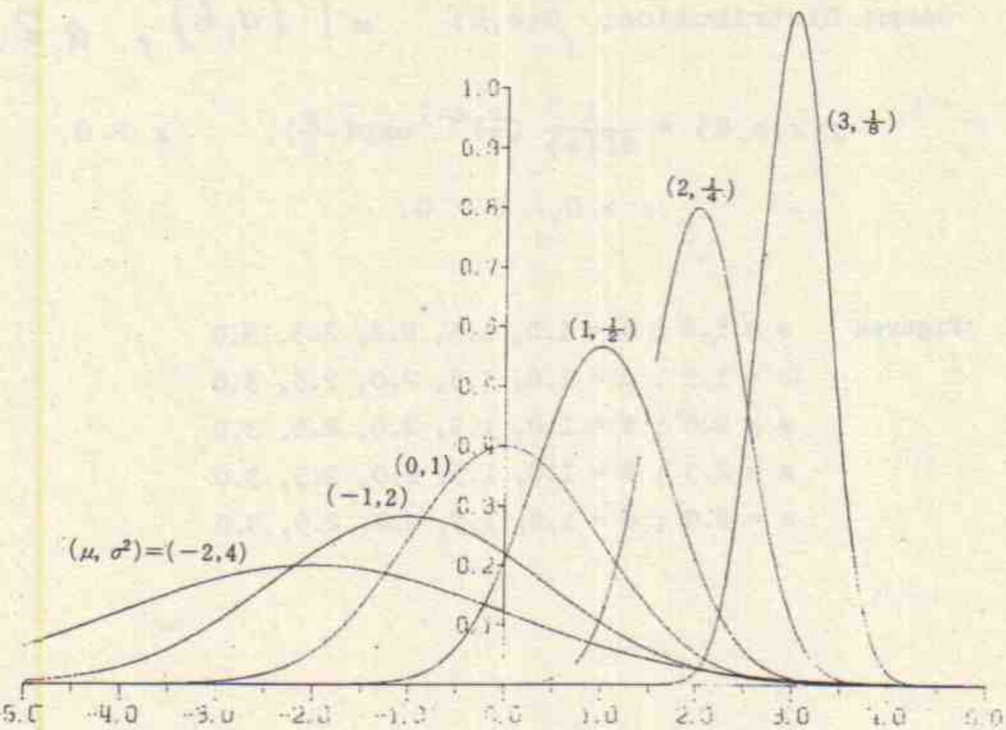
$(\mu,\sigma^2) = (-2,4), (-1,2), (0,1), (1,\frac{1}{2}), (2,\frac{1}{4}), (3,\frac{1}{8})$

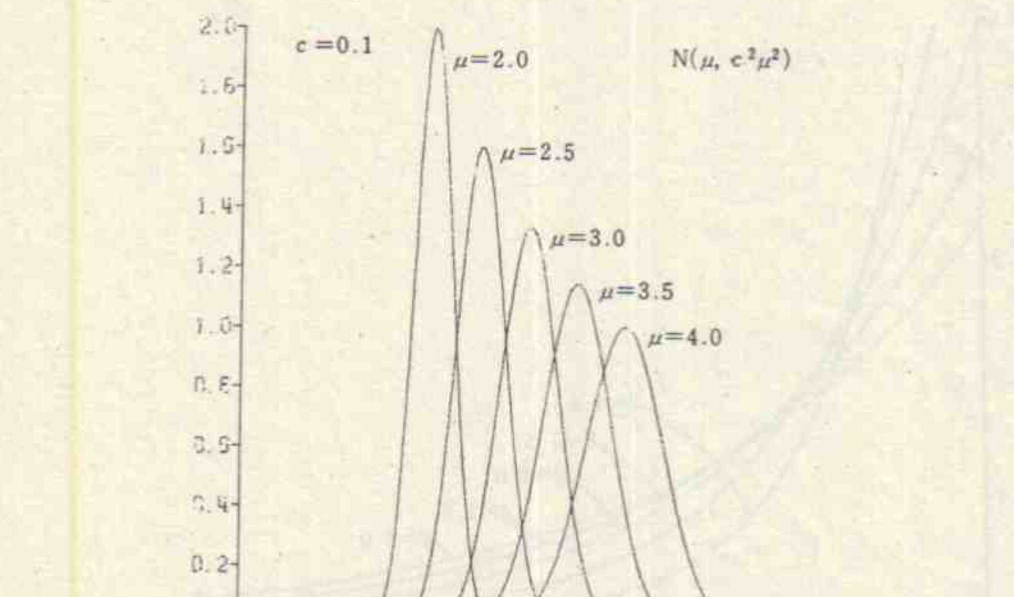
$N(\mu,c^2\mu^2)$; $c = 0.1$; $\mu = 2.0, 2.5, 3.0, 3.5, 4.0$

$c = 0.3$; $\mu = 1.0, 1.5, 2.0, 2.5, 3.0$

$c = 0.5$; $\mu = 0.5, 1.0, 1.5, 2.0, 2.5$




(3, 1/8)
(2, 1/4)
(1, 1/2)
(0,1)
(−1,2)
(μ, σ²)=(−2,4)

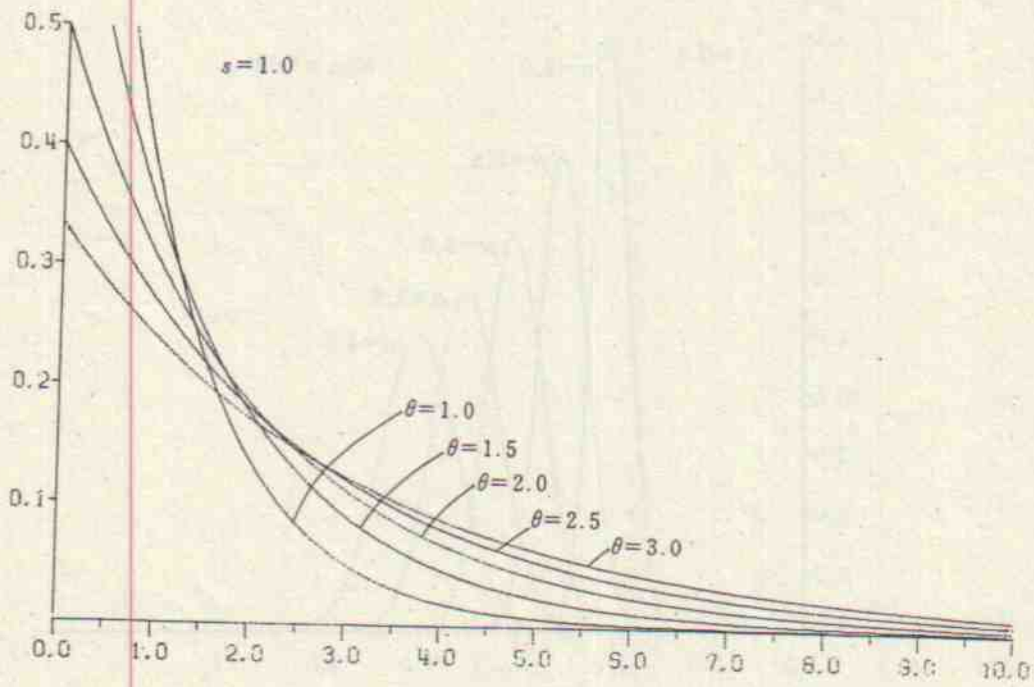

c =0.1
μ=2.0
N(μ, c²μ²)
μ=2.5
μ=3.0
μ=3.5
μ=4.0

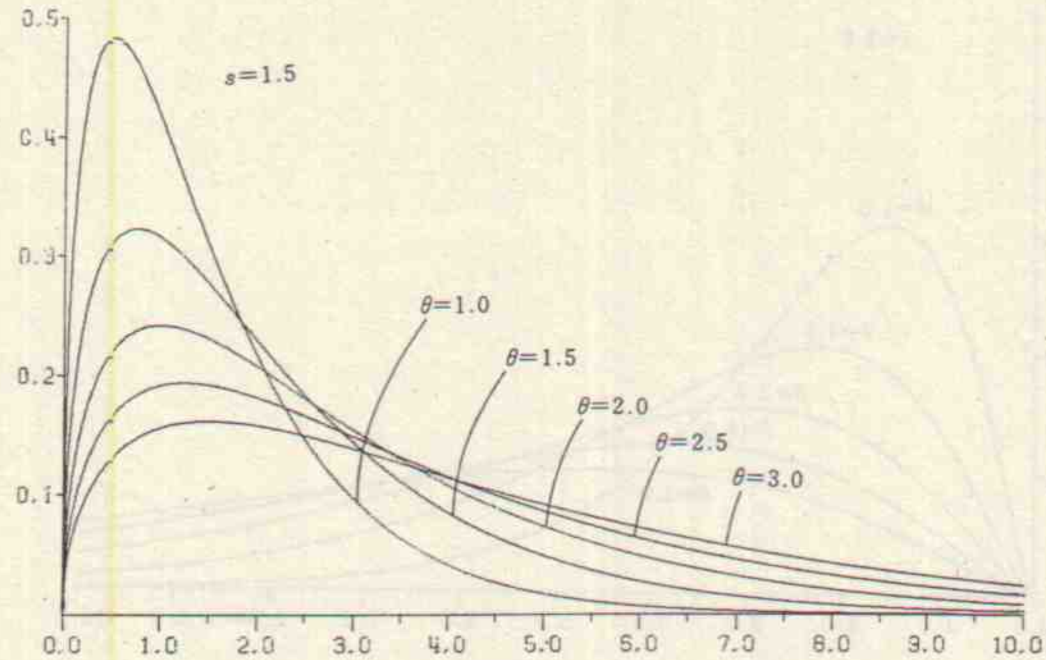
3. Gamma Distribution: $G(s,\theta)$ $= \Gamma(a,b),\ a \mathrel{\hat{=}} \vartheta,\ b \mathrel{\hat{=}} s$

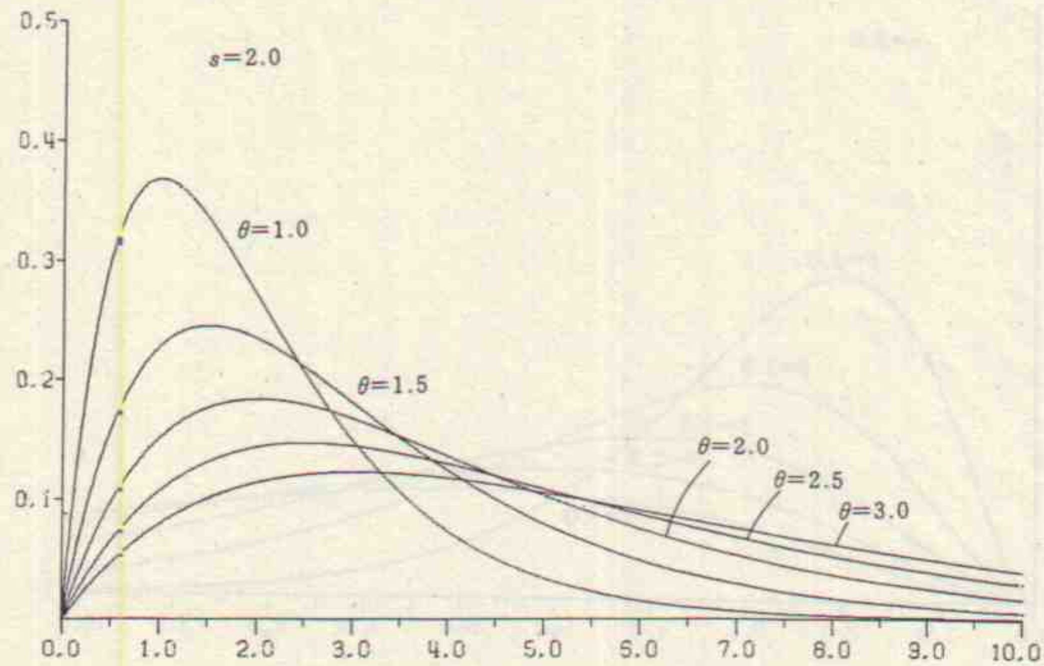
$$f(x;s,\theta) = \frac{1}{\theta\Gamma(s)} \left(\frac{x}{\theta}\right)^{s-1} \exp\left(-\frac{x}{\theta}\right), \qquad x > 0,$$

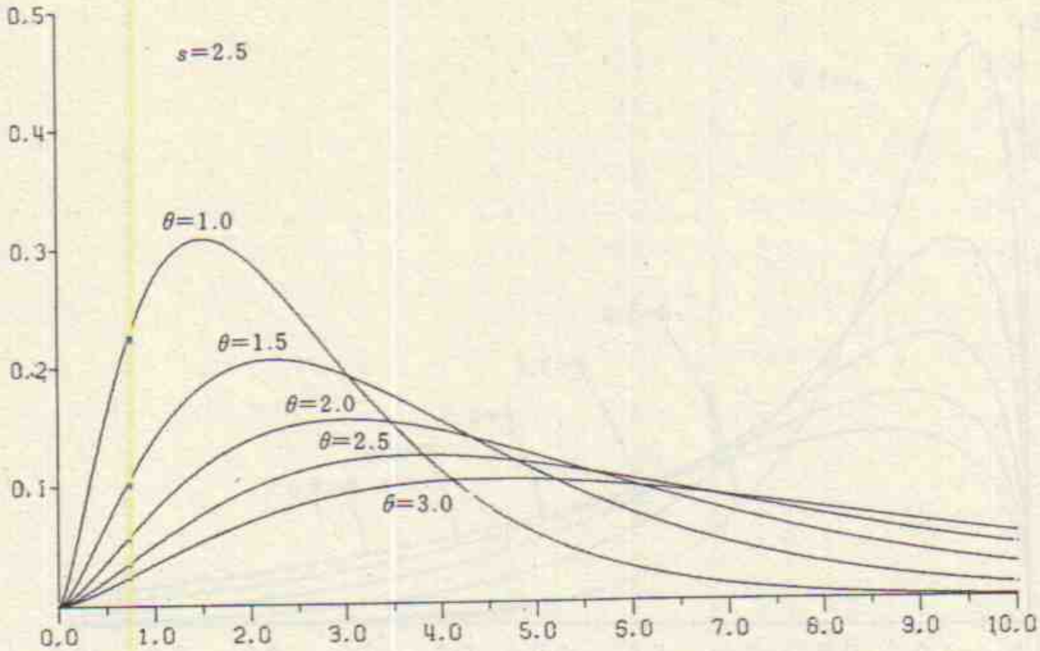
$$s > 0, \quad \theta > 0.$$

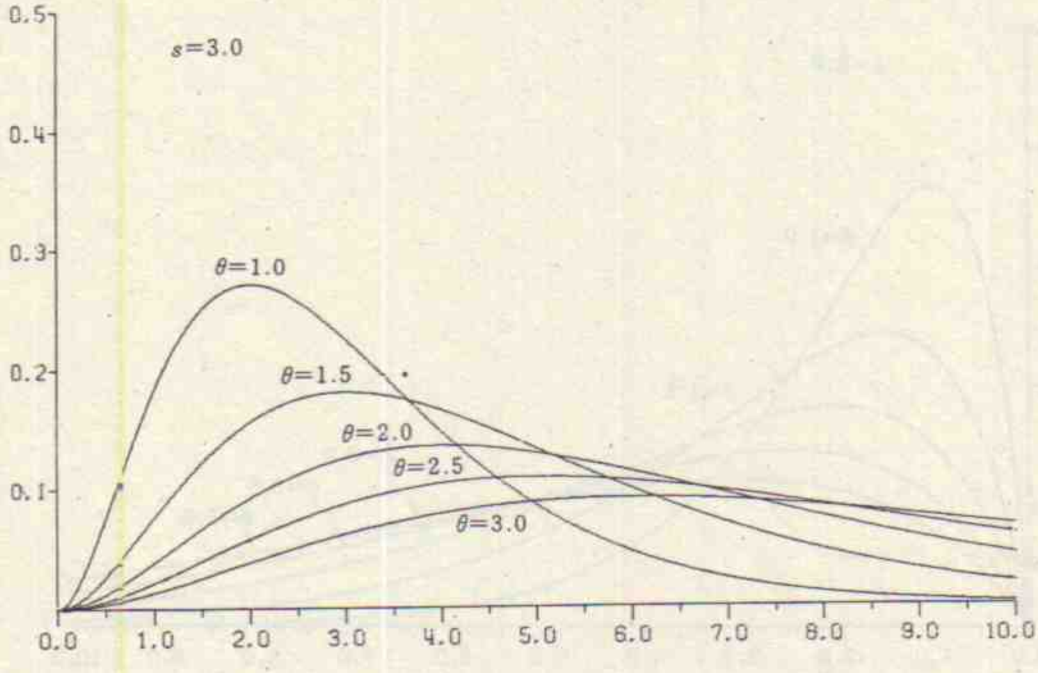
Figures $s = 1.0$; $\theta = 1.0, 1.5, 2.0, 2.5, 3.0$
$s = 1.5$; $\theta = 1.0, 1.5, 2.0, 2.5, 3.0$
$s = 2.0$; $\theta = 1.0, 1.5, 2.0, 2.5, 3.0$
$s = 2.5$; $\theta = 1.0, 1.5, 2.0, 2.5, 3.0$
$s = 3.0$; $\theta = 1.0, 1.5, 2.0, 2.5, 3.0$




s=1.5
θ=1.0
θ=1.5
θ=2.0
θ=2.5
θ=3.0
0.5
0.4
0.3
0.2
0.1
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0


s=2.0
θ=1.0
θ=1.5
θ=2.0
θ=2.5
θ=3.0
0.5
0.4
0.3
0.2
0.1
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0


s=2.5
θ=1.0
θ=1.5
θ=2.0
θ=2.5
θ=3.0
0.5
0.4
0.3
0.2
0.1
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0

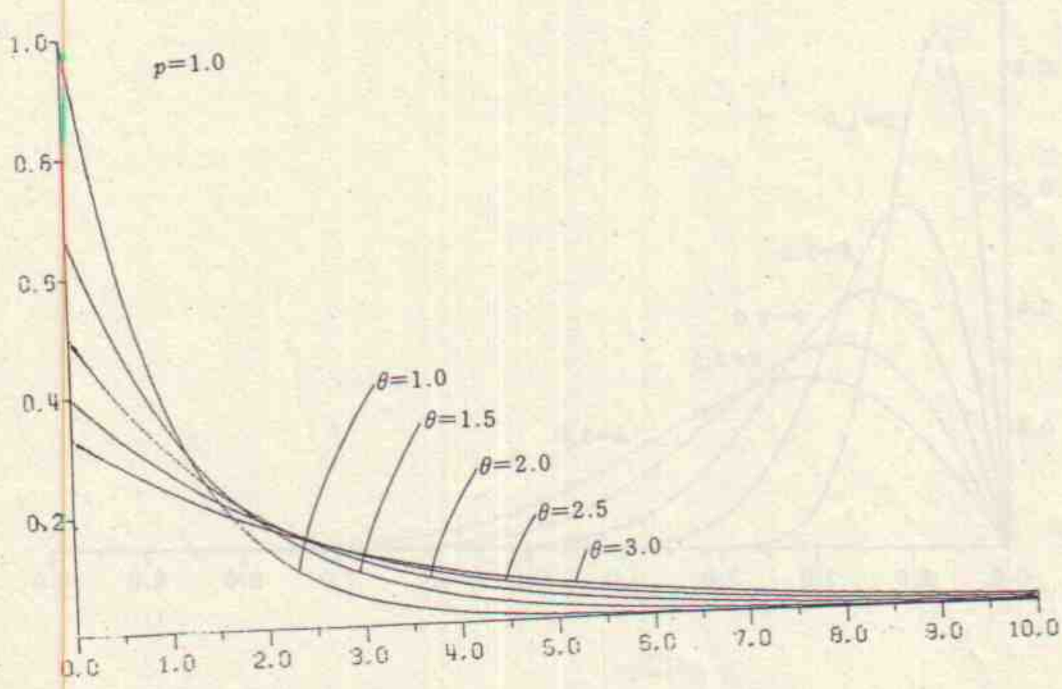

s=3.0
θ=1.0
θ=1.5
θ=2.0
θ=2.5
θ=3.0
0.5
0.4
0.3
0.2
0.1
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0

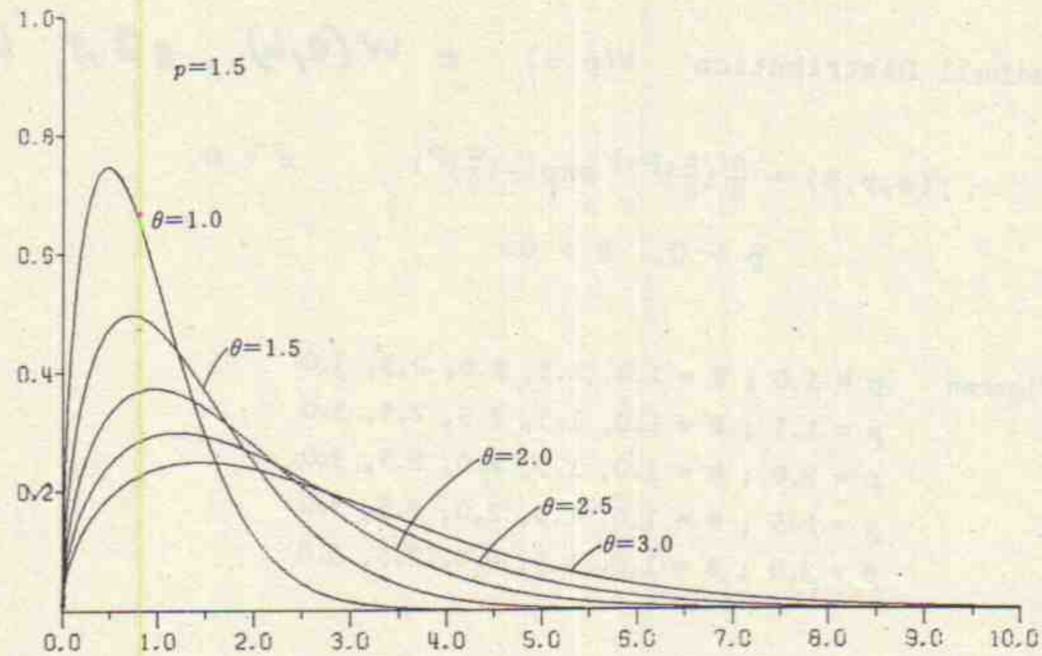
4. Weibull Distribution: $W(p,\theta)$ $= W(a,b)$, $a \hat{=} \vartheta_1$, $b \hat{=} p$

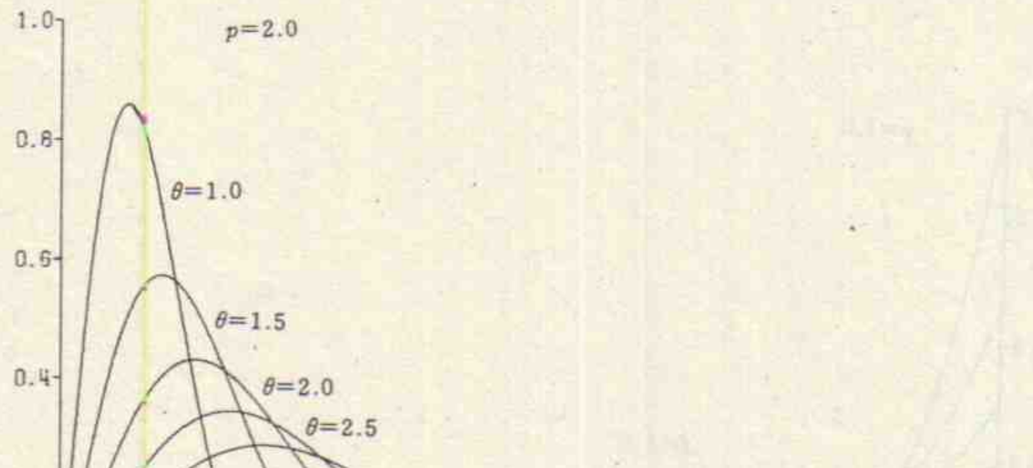
$$f(x;p,\theta) = \frac{p}{\theta}\left(\frac{x}{\theta}\right)^{p-1} \exp\{-\left(\frac{x}{\theta}\right)^{p}\}, \qquad x > 0,$$

$$p > 0, \quad \theta > 0.$$

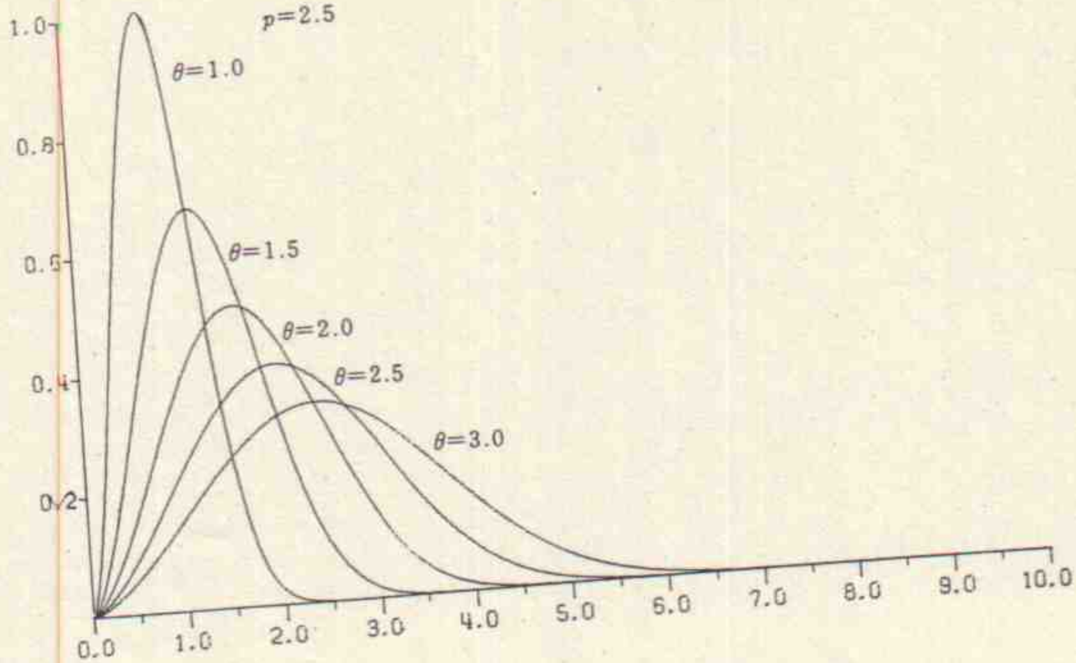
Figures $p = 1.0$; $\theta = 1.0, 1.5, 2.0, 2.5, 3.0$
$p = 1.5$; $\theta = 1.0, 1.5, 2.0, 2.5, 3.0$
$p = 2.0$; $\theta = 1.0, 1.5, 2.0, 2.5, 3.0$
$p = 2.5$; $\theta = 1.0, 1.5, 2.0, 2.5, 3.0$
$p = 3.0$; $\theta = 1.0, 1.5, 2.0, 2.5, 3.0$

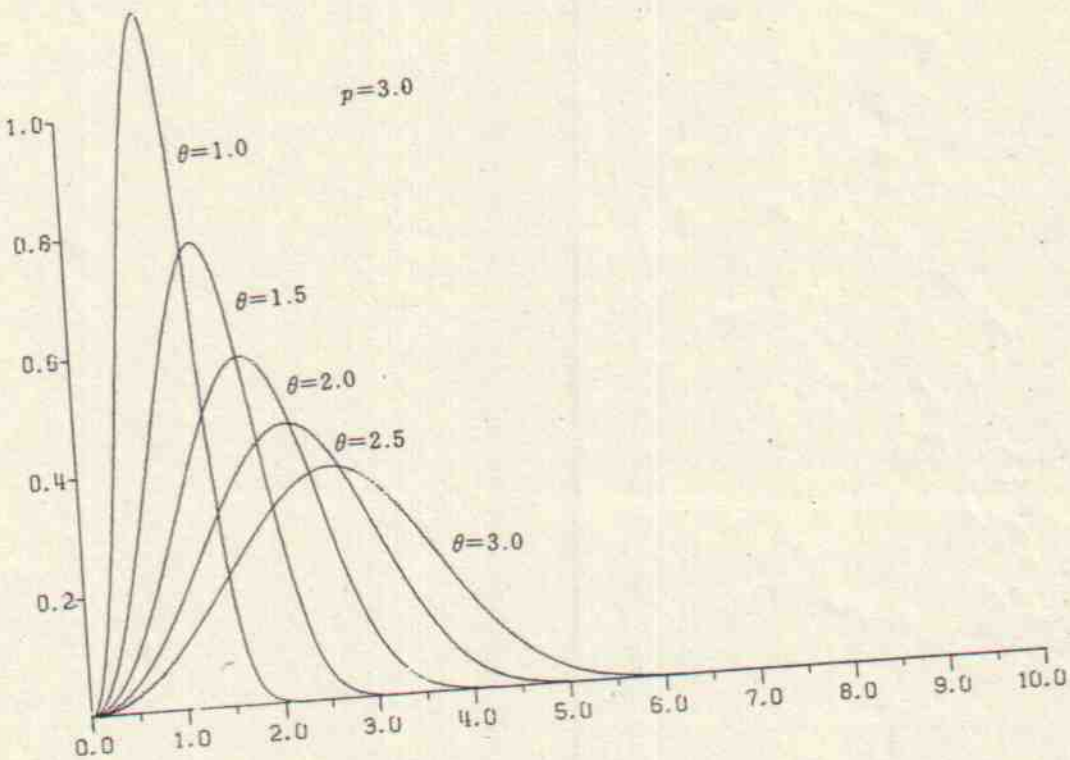



p=1.5
θ=1.0
θ=1.5
θ=2.0
θ=2.5
θ=3.0
1.0
0.8
0.6
0.4
0.2
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0


p=2.0
θ=1.0
θ=1.5
θ=2.0
θ=2.5
θ=3.0
1.0
0.8
0.6
0.4
0.2
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0

10


p=2.5
θ=1.0
θ=1.5
θ=2.0
θ=2.5
θ=3.0
1.0
0.8
0.6
0.4
0.2
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0


p=3.0
θ=1.0
θ=1.5
θ=2.0
θ=2.5
θ=3.0
1.0
0.8
0.6
0.4
0.2
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0