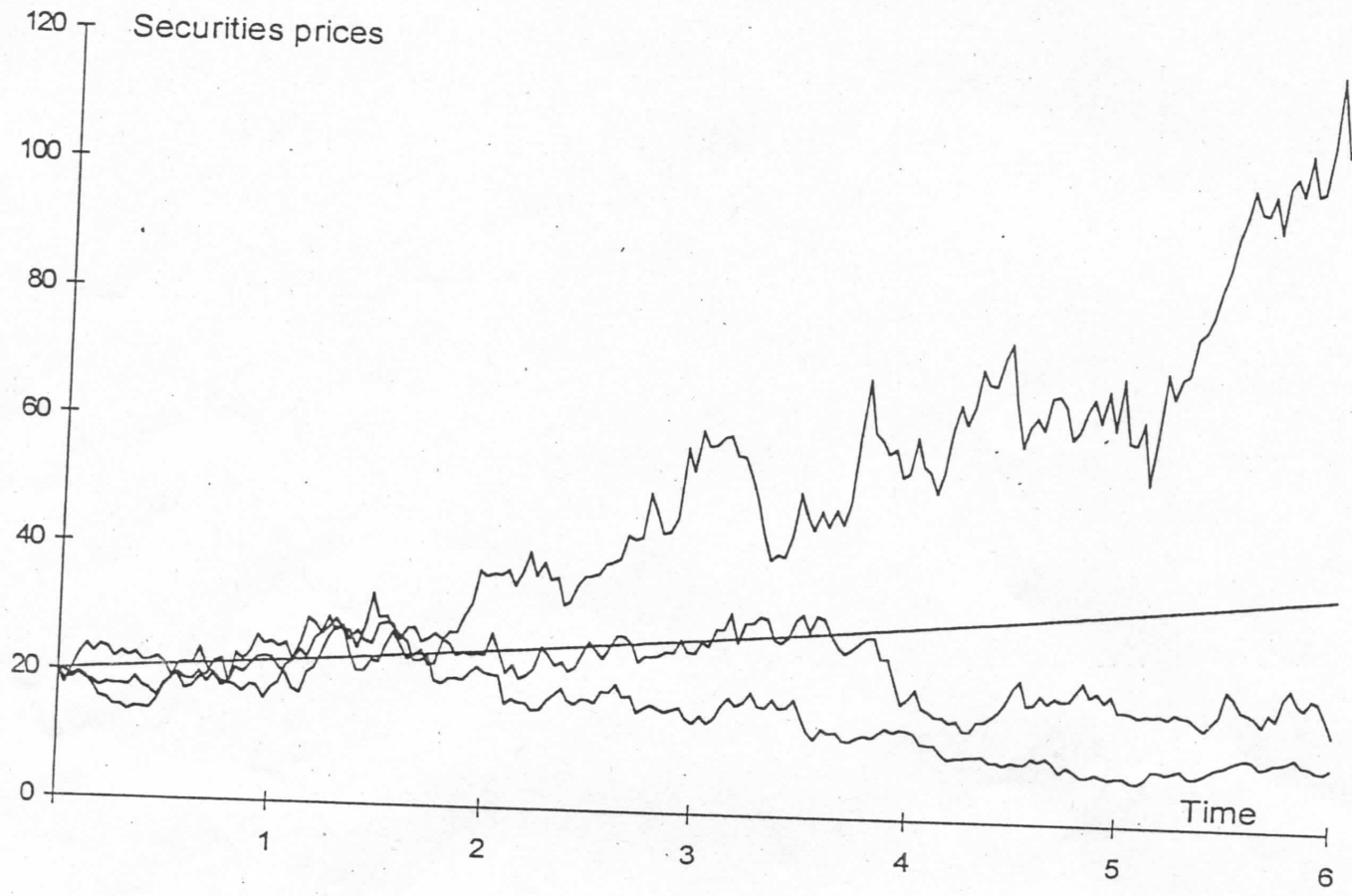FIG. 1.5. Securities price processes in the Black–Scholes model

Figure 1.5 shows three paths of the stock price and one path of the value of the money market account in the Black–Scholes model. Both the stock and the money market account start at an initial value of 20. The interest rate is $r = 0.1$, the instantaneous expected rate of return to the stock is $\mu = 0.2$, and the volatility of the stock is $\sigma = 0.4$.
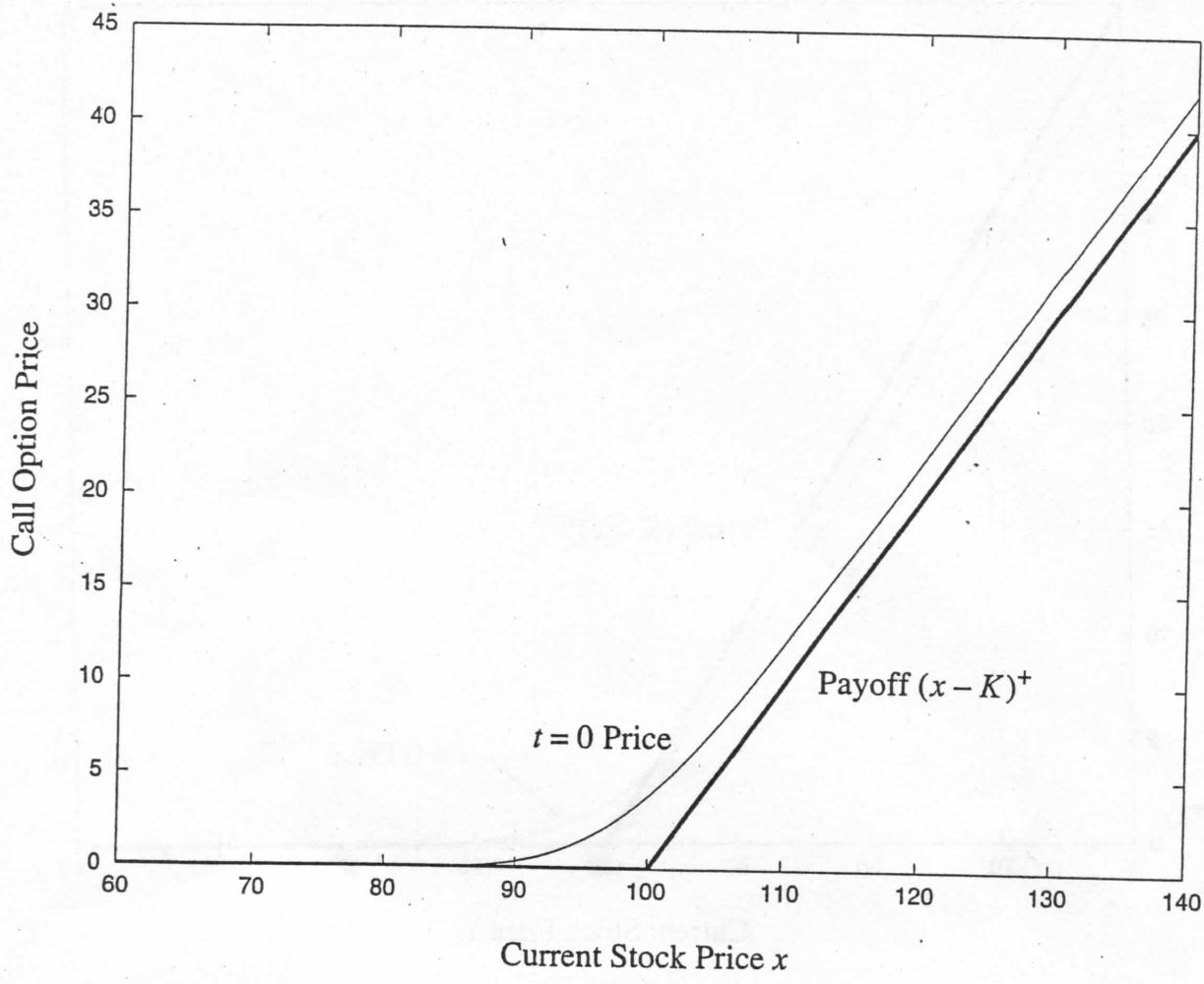
□

$V^{\text{Call}}_{(0,x)}$

Figure 1.2. Black–Scholes call option pricing function at time $t = 0$, with $K = 100$, $T = 0.5$, $\sigma = 0.1$, and $r = 0.04$.
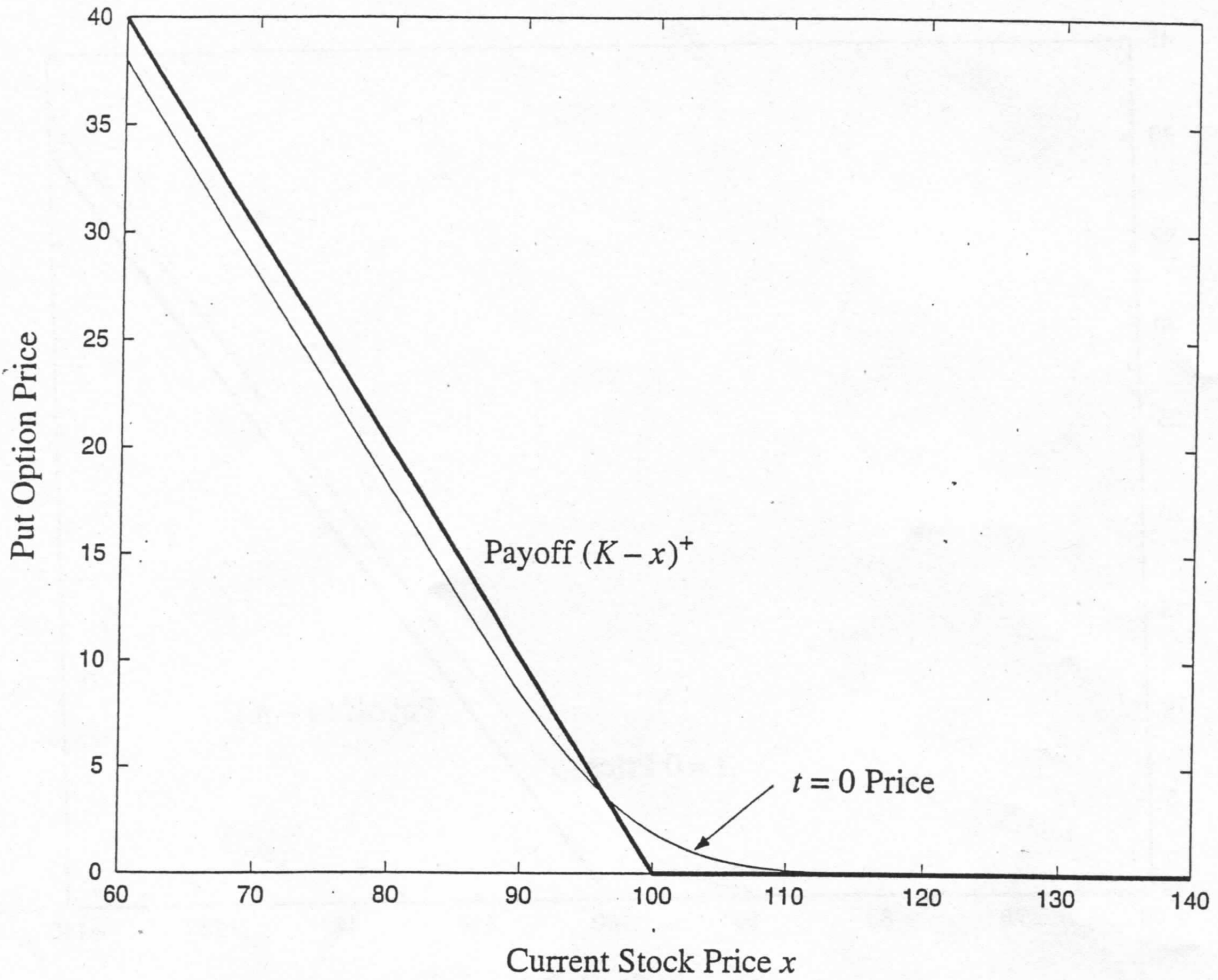
$$V^{Put}_{(0,x)}$$

Figure 1.3. Black–Scholes put option pricing function at time $t = 0$, with $K = 100$, $T = 0.5$, $\sigma = 0.1$, and $r = 0.04$.

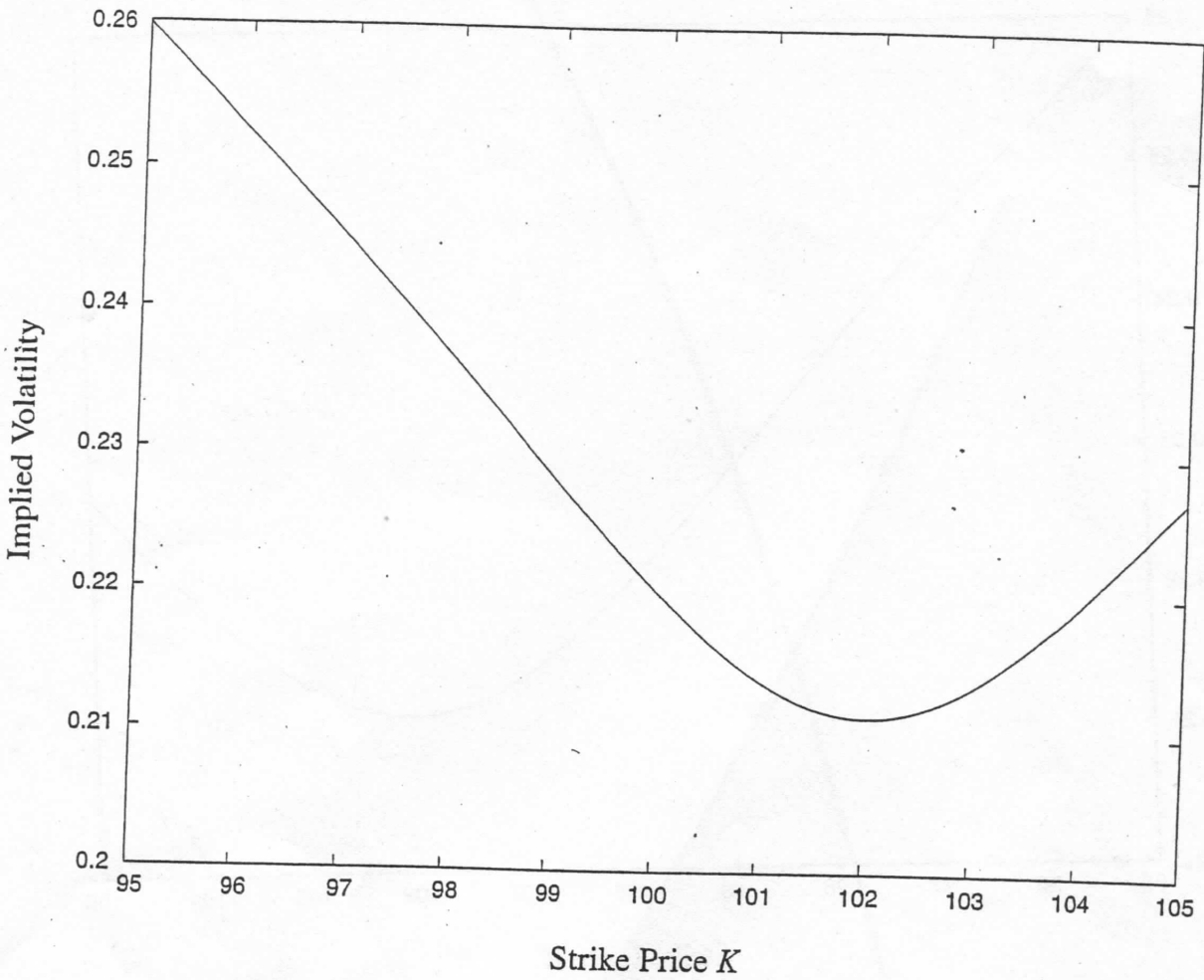$$\left[\ V^{put}_{(0,x)} = V^{call}_{(0,x)} + Ke^{-rT} - x\ \right]$$

Figure 2.1. Illustrative smile curve of implied volatilities from European options with the same time to expiration. The current stock price is $x = 100$, which is close to the minimum point. In fact, the minimum is at $xe^{r(T-t)} \approx 102$.