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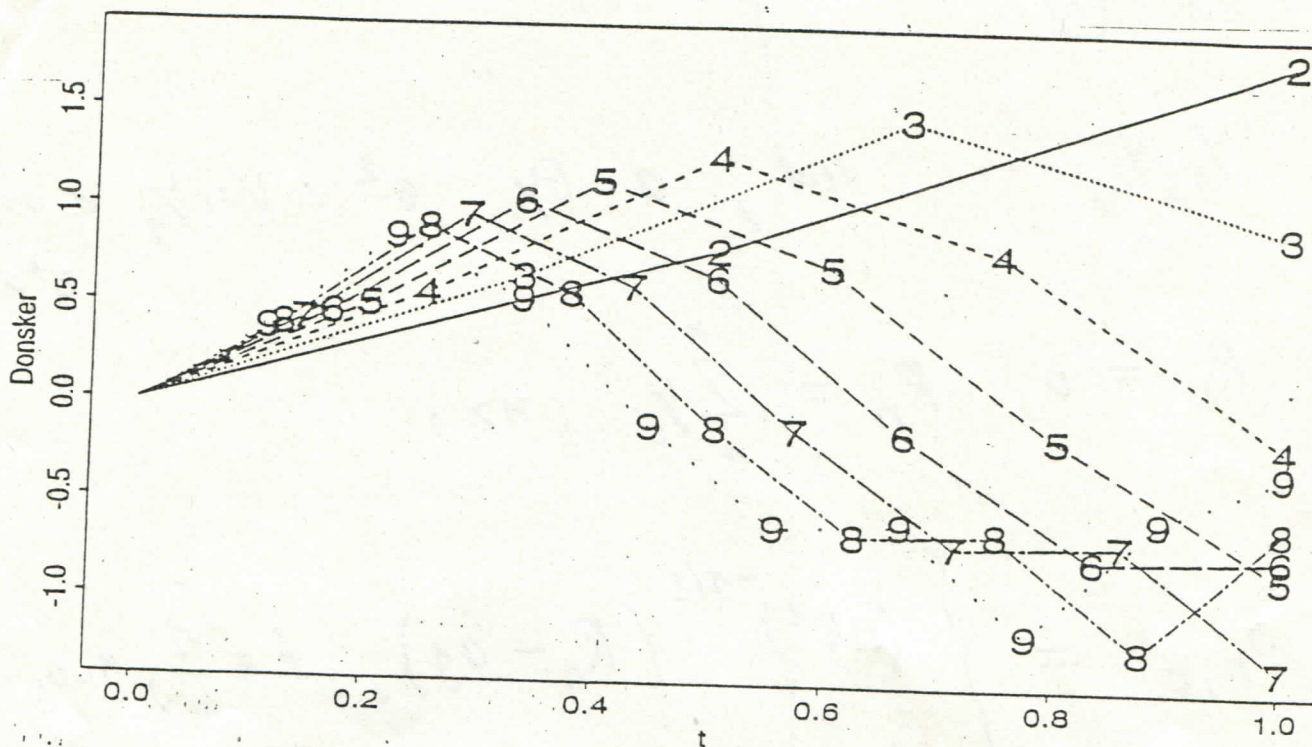


Figure 1.3.11 Sample paths of the process S_n for one sequence of realizations $Y_1(\omega), \dots, Y_9(\omega)$ and $n = 2, \dots, 9$.

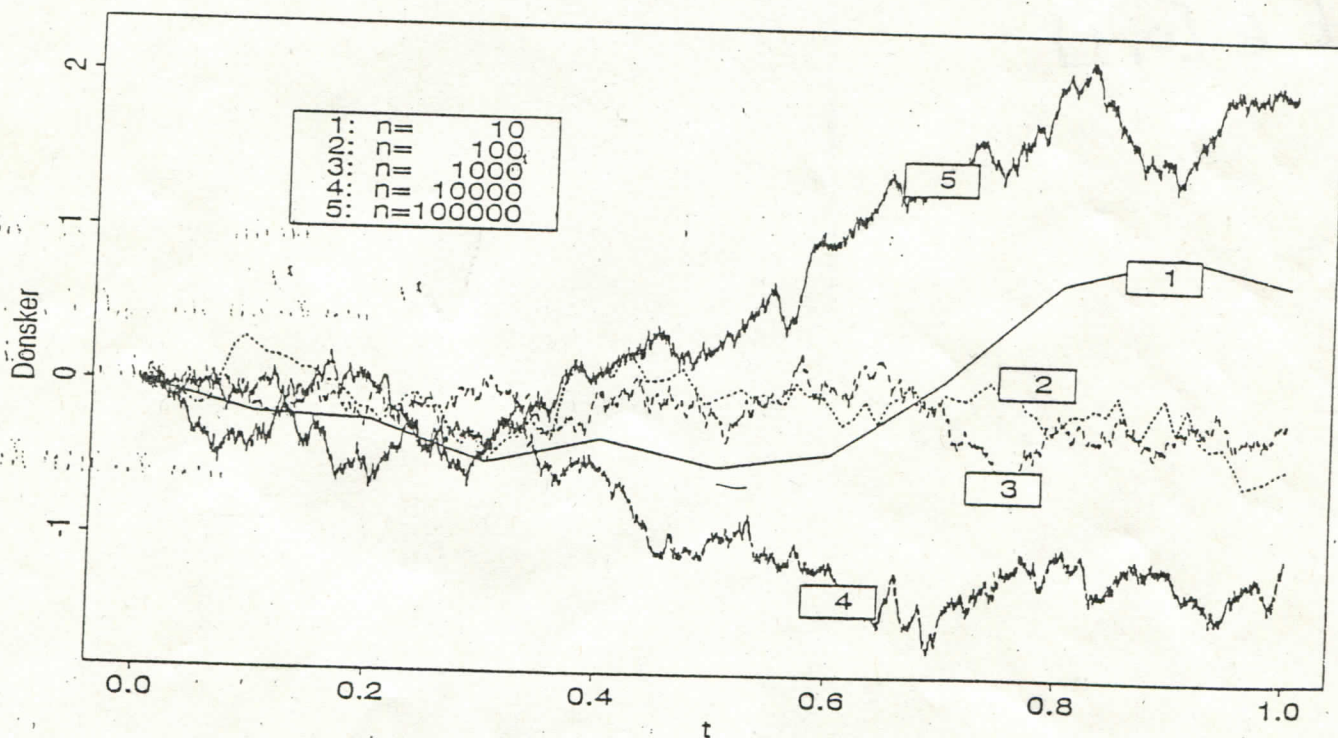


Figure 1.3.12 Sample paths of the process S_n for different n and the same sequence of realizations $Y_1(\omega), \dots, Y_{100,000}(\omega)$.

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Y_1, Y_2, \dots iid, $a = EY_1$, $\sigma^2 = \text{Var} Y_1 < \infty$,
 $\sigma^2 > 0$

$$R_0 = 0, \quad R_n = \sum_{i=1}^n Y_i, \quad n \geq 1$$

$$S_{n,t} = \begin{cases} (n\sigma^2)^{-1/2} (R_j - a_j), & t = \frac{j}{n}, j=0, \dots, n \\ \text{linear interpoliert, sonst} \end{cases}$$

$$t \in [0, 1]$$

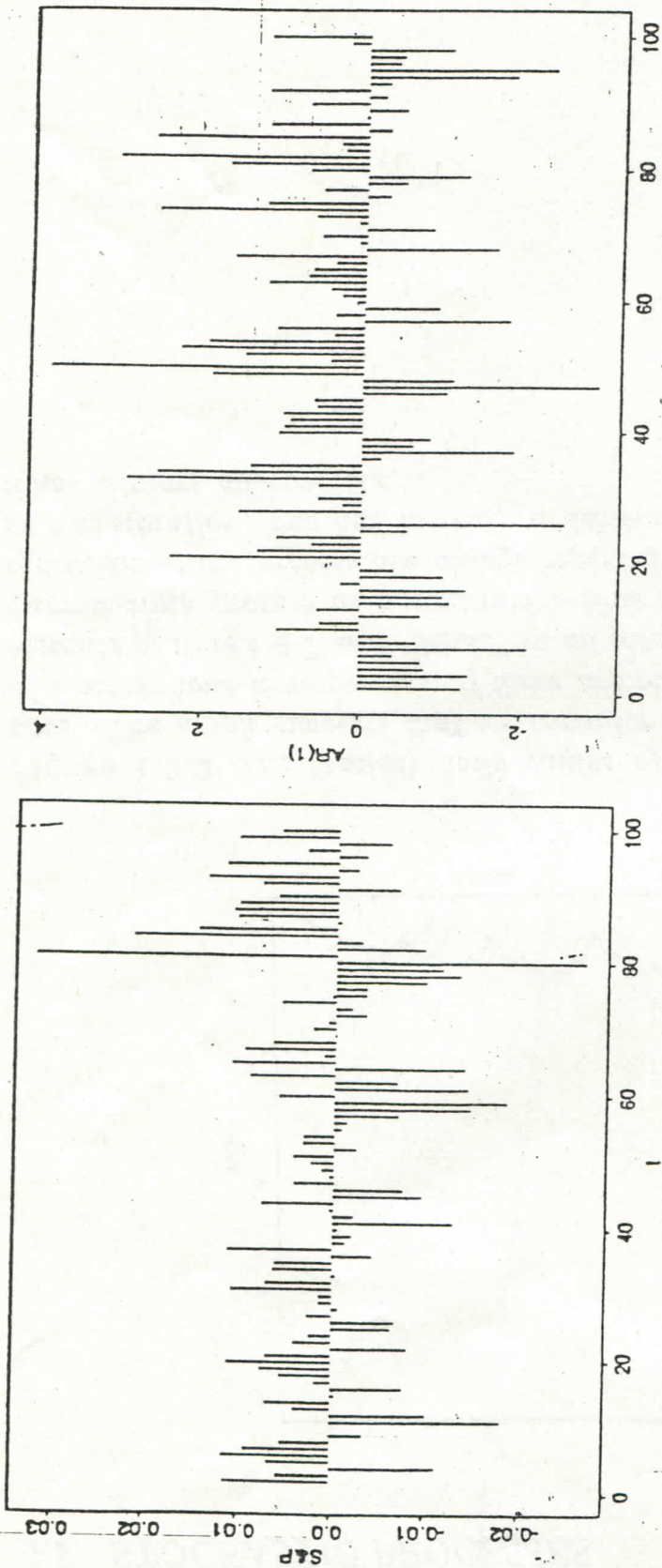


Figure 1.2.4 Two time series X_t , $t = 1, \dots, 100$. Left: 100 successive daily log-returns of the S&P index; see Figure 1.1.4. Right: a simulated sample path of the autoregressive process $X_t = 0.5X_{t-1} + Z_t$, where Z_t are iid $N(0,1)$ random variables; see Example 1.2.3.

1.2. STOCHASTIC PROCESSES

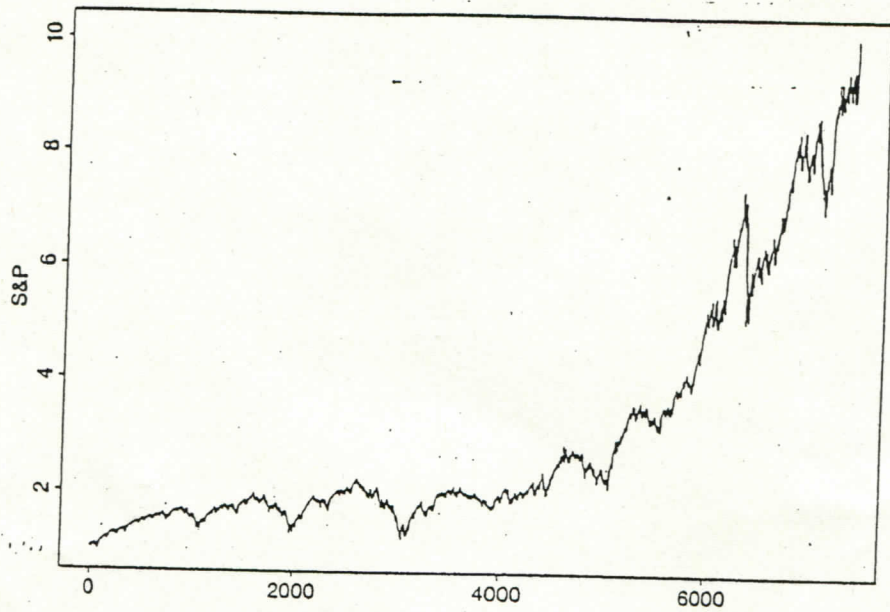


Figure 1.2.2 The (scaled) daily values of the S&P index over a period of 7,422 days. The graph suggests that we consider the S&P time series as the sample path of a continuous-time process. If there are many values in a time series such that the instants of time $t \in T$ are "dense" in an interval, then one may want to interpret this discrete-time process as a continuous-time process. The sample paths of a real-life continuous-time process are always reported at discrete instants of time. Depending on the situation, one has to make a decision which model (discrete- or continuous-time) is more appropriate.