

$T_{N_k}(\omega) = T_n(x_1, \dots, x_n)$. We shall say that a statistic T_n is asymptotically normal if there exists a positive function $\sigma^2(\theta)$, $\theta \in \Theta$, such that for each $\theta \in \Theta$,

$$\lim_{n \rightarrow \infty} \mathbf{P}_\theta \left(n^{1/2}(T_n - \theta) < x \right) = \Phi \left(\frac{x}{\sigma(\theta)} \right), \quad x \in \mathbf{R}^1,$$

where $\Phi(x)$ is the standard normal d.f. The property of asymptotic normality under rather general conditions is inherent in many statistics. As an example, we can mention the statement on the asymptotic normality of the maximum likelihood estimators from [6] which is possibly most known. The transformation of this property under a random sample size is described by the following statement which improves some statements from [7].

THEOREM 3. *Let a statistic T_n be asymptotically normal and let (3) hold. Then for the existence for each $\theta \in \Theta$ of a d.f. $F(x, \theta)$ such that $\mathbf{P}_\theta(k^{1/2}(T_{N_k} - \theta) < x) \implies F(x, \theta)$ ($k \rightarrow \infty$), it is necessary and sufficient that there exists a family of d.f.'s $\mathcal{G} = \{G(x; \theta): \theta \in \Theta\}$ satisfying the following conditions:*

- 1) $G(x; \theta) = 0$, $x < 0$, $\theta \in \Theta$;
- 2) $F(x; \theta) = \int_0^\infty \Phi(u^{1/2}x/\sigma(\theta)) dG(u; \theta)$, $x \in \mathbf{R}^1$, $\theta \in \Theta$;
- 3) $\mathbf{P}_\theta(N_k < kx) \implies G(x; \theta)$ ($k \rightarrow \infty$), $\theta \in \Theta$.

Moreover, if the d.f.'s of the r.v.'s N_k do not depend on θ , then the d.f. G also does not depend on θ , i.e., the family \mathcal{G} consists of a single element.

Proof. By virtue of the identifiability of the family of scale mixtures of normal laws, the proof reduces to Theorem 1 with $b_k = k^{-1/2}$, $S_k = T_k - \theta$, and $a_k = 0$, $k \geq 1$. The main steps of the proof are similar to those of the preceding theorem. The theorem is proved.

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OPTIMAL UNBIASED ESTIMATORS IN ADDITIVE MODELS WITH BOUNDED ERRORS ARE DETERMINISTIC*

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Abstract. In an additive model $X = \vartheta + \varepsilon$, $\vartheta \in \Theta \subset \mathbf{R}^k$, let the errors ε have a compactly supported but otherwise arbitrary known joint distribution. Let g be a uniformly minimum variance unbiased estimator for its own expectation $\gamma(\vartheta)$. We show that under mild regularity conditions, g

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is deterministic: for every $\vartheta \in \Theta$, $g(X) = \gamma(\vartheta)$ almost surely. Our proof uses a lemma on entire quotients of Fourier transforms which might be of independent interest.

Key words. characteristic function, entire function, exponential type, Fourier transform, linear model, location parameter, shift model, uniformly minimum variance unbiased estimator

1. Introduction and results. Given a statistical model $\mathcal{P} = \{P_\vartheta: \vartheta \in \Theta\}$ consisting of probability measures P_ϑ on a common measurable space, it is often difficult to decide which functions $\gamma: \Theta \rightarrow \mathbf{R}$, if any, possess a uniformly minimum variance unbiased (UMVU) estimator. It is therefore of interest to know criteria which for some models permit us to conclude that, in fact, only trivial UMVU estimators exist.

For arbitrary models \mathcal{P} , Bahadur reduced the existence question for bounded nontrivial UMVU estimators to the question of whether there are any UMVU indicator variables (see, e.g., §2 of [10]). This is not always an essential simplification of the problem (see, e.g., §2.2 below, where the assumption “ g is $\{0, 1\}$ -valued” would not be of help).

Thus far, only a short list of interesting, natural examples of models admitting only trivial UMVU estimators seems to be available from the literature. The first example, due to Lehmann and Scheffé [7, pp. 325–327] and completed in [4, Thm. 5.1], has $\Theta = \mathbf{R}$ and $P_\vartheta = n$ -fold product of the uniform distribution over the interval $[\vartheta, \vartheta + 1]$. A discrete version can be found in [6, p. 79]. Examples where \mathcal{P} is an algebraic exponential family can be deduced from [5, Thm. S.3.1]. Unfortunately, no concrete examples are given in [5]. This has been done, according to [6, p. 89], by Unni [12]. Sapozhnikov [11] gave an interesting example occurring when sampling from a finite population with replacement, leading to binomial distributions with a discrete parameter space. Sapozhnikov’s method is extended from discrete sample space to more general ones in [3], where, however, no natural example is treated.

In this paper, we consider additive models with bounded errors. We show that every sufficiently regular UMVU estimator is deterministic (in the sense of Definition 2 below). In reasonable cases, it follows that the function γ being estimated is constant (see Remark 2 below).

DEFINITION 1. $\mathcal{P} = \{P_\vartheta: \vartheta \in \Theta\}$ is an additive model if for some positive integer k , we have $\Theta \subset \mathbf{R}^k$ and $P_\vartheta = P(\cdot - \vartheta)$ for some probability measure P on the Borel sets of \mathbf{R}^k . P is called the (joint) error distribution.

In other words, we consider observing $X = \vartheta + \varepsilon$, where $\vartheta \in \Theta \subset \mathbf{R}^k$ is an unknown parameter and the joint distribution P of the errors ε is known.

DEFINITION 2. A Borel function $g: \mathbf{R}^k \rightarrow \mathbf{R}$ is called deterministic (with respect to \mathcal{P}) if for some $\gamma: \Theta \rightarrow \mathbf{R}$ and every $\vartheta \in \Theta$

$$g(x) = \gamma(\vartheta) \quad P_\vartheta\text{-almost surely.}$$

THEOREM. Let \mathcal{P} be an additive model. Assume that the error distribution P has compact support. Let g be a UMVU estimator for its own expectation. If for each $\vartheta \in \Theta$, g is bounded on the support of P_ϑ , then g is deterministic.

Remark 1. Observe that no restriction is imposed on $\Theta \subset \mathbf{R}^k$. Thus the theorem applies in particular to every linear model with bounded errors by taking Θ as a suitable linear subspace of \mathbf{R}^k . The special choice $\Theta = \{(\alpha, \dots, \alpha): \alpha \in \mathbf{R}\}$ and $P = \mu \otimes \dots \otimes \mu$ for some probability measure μ on \mathbf{R} leads to models for independent and identically distributed observations from a full location family considered in [4]; see Remark 4 below.

Remark 2. If in an additive model \mathcal{P} the distribution P has a density f with respect to Lebesgue measure and if Θ is connected, then any deterministic g is in fact constant with respect to \mathcal{P} , i.e., the γ in Definition 2 has to be a constant function.

To prove this, note that

$$\lim_{\vartheta \rightarrow \vartheta_0} \int f(x - \vartheta) f(x - \vartheta_0) dx = \int f^2(x - \vartheta_0) dx > 0.$$

(Use the continuity of translation in L^2 if $\int f^2(x) dx < \infty$. Otherwise, first consider $f \wedge n$ and then let n tend to infinity.) Hence for $|\vartheta - \vartheta_0|$ sufficiently small, $\{f(\cdot - \vartheta) > 0 \cap \{f(\cdot - \vartheta_0) > 0\}$ has positive Lebesgue measure. This implies $\gamma(\vartheta) = \gamma(\vartheta_0)$, i.e., γ is locally constant.

Remark 3. Obvious examples of nonconstant functions γ in additive models \mathcal{P} admitting deterministic UMVU estimators are as follows:

- (i) P is the Dirac measure and γ and Θ are arbitrary;
- (ii) $k = 1$, P is concentrated on the integers, γ is an arbitrary 1-periodic function, and Θ is arbitrary;
- (iii) $k = 3 \cdot 2$ with $P = \mu \otimes \mu \otimes \mu$, μ is uniform distribution on the unit circle (not disk) in the plane, and $\Theta = \{(\alpha, \alpha, \alpha) : \alpha \in \mathbf{R}^2\}$ is arbitrary.

An attempt to describe all such examples does not appear to be worthwhile.

Remark 4. The regularity condition imposed on g is rather weak and, e.g., satisfied by any continuous g . This should be compared with the results due to Bondesson [4], most of which apply to models \mathcal{P} with noncompactly supported P but impose global growth conditions or even structural assumptions on g (see [4, Thm. 3.3 and 5.2]).

In the case when \mathcal{P} is a full shift model (i.e., $\Theta = \mathbf{R}^k$) and is sufficiently regular, we are able to drop the regularity condition imposed on g .

COROLLARY. *Let \mathcal{P} be an additive model with $\Theta = \mathbf{R}^k$. Assume that the error distribution P has a compactly supported density f with respect to Lebesgue measure. Assume further that the set $\{x \in \mathbf{R}^k : f(x) > \delta\}$ has nonempty interior for some $\delta > 0$. Let g be a UMVU estimator for its own expectation. Then g is constant almost surely with respect to \mathcal{P} .*

Remark 5. Our results contain the Lehmann–Scheffé–Bondesson example mentioned above, provided that only locally bounded estimators are considered in the case $n \geq 2$. This follows from Remarks 1 and 2 and the corollary. Without any restriction on the estimators, our results are easily seen to contain Lehmann’s discrete example mentioned above and also Theorem 3.4 in [4].

Remark 6. Our proof of the theorem given below shows in fact that for fixed $\vartheta_0 \in \Theta$ it suffices to assume that g is locally minimum variance unbiased in ϑ_0 and bounded on the support of P_{ϑ_0} in order to conclude that g is almost surely constant with respect to P_{ϑ_0} . We do not know whether a similar strengthening of the corollary is possible.

2. Proofs.

2.1. Preliminaries for the proof of the theorem. An entire function f is of exponential type (or, more precisely, of exponential type τ) if

$$\tau = \limsup_{r \rightarrow \infty} \frac{\log M(r, f)}{r} < \infty,$$

where $M(r, f) = \max\{|f(\zeta)| : |\zeta| = r\}$ (see [2, p. 8]). The proof of the theorem is based on the following lemma. It uses the fact that a measure μ with compact support $\text{supp } \mu$ has a Fourier transform $\widehat{\mu}(\zeta) = \int e^{i\zeta x} d\mu(x)$ which is an entire function of exponential type growing quite regularly.

LEMMA 1. *Let μ, ν be complex measures on the Borel sets of \mathbf{R} having compact support. Assume that (i) the convex hull of $\text{supp } \nu$ is a subset of the convex hull of $\text{supp } \mu$, (ii) $\widehat{\nu}/\widehat{\mu}$ is an entire function. Then $\widehat{\nu}/\widehat{\mu}$ is an entire function of exponential type zero.*

Proof. We need some definitions and results from the theory of entire functions. The indicator function h_f of an entire function of exponential type is given by

$$h_f(\phi) = \limsup_{r \rightarrow \infty} \frac{\log |f(re^{i\phi})|}{r} \quad (0 \leq \phi \leq 2\pi)$$

(see [2, p. 66]). h_f is (the angular part of) the supporting function of some nonempty compact convex set $K(f) \subset \mathbf{C}$, the indicator diagram of f , i.e.,

$$h_f(\phi) = \max \left\{ \text{Re}(\zeta e^{-i\phi}) : \zeta \in K(f) \right\}$$

(see [2, pp. 70–72]). f is of *completely regular growth* if there is a set $E \subset (0, \infty)$ of linear density zero such that

$$\lim_{\substack{r \rightarrow \infty \\ r \notin E}} \frac{\log |f(re^{i\phi})|}{r} = h_f(\phi) \quad \text{uniformly in } \phi$$

(see [8, pp. 139 and 96]).

Now let $[a, b]$ ($[c, d]$) be the convex hull of $\text{supp } \mu$ ($\text{supp } \nu$, respectively). It follows from [9, Thm. 3] that $\widehat{\mu}$ and $\widehat{\nu}$ are entire functions of exponential type and

$$(1) \quad h_{\widehat{\mu}}\left(\frac{\pi}{2}\right) = -a, \quad h_{\widehat{\mu}}\left(\frac{3\pi}{2}\right) = b, \quad h_{\widehat{\nu}}\left(\frac{\pi}{2}\right) = -c, \quad h_{\widehat{\nu}}\left(\frac{3\pi}{2}\right) = d.$$

Theorem 11 in [8, p. 251] states that $\widehat{\mu}$ and $\widehat{\nu}$ are of completely regular growth and that their indicator diagrams are intervals on the imaginary axis. Because of (1), we have $K(\widehat{\mu}) = [-ib, -ia]$ and $K(\widehat{\nu}) = [-id, -ic]$. The assumption $\text{supp } \nu \subset \text{supp } \mu$ implies $K(\widehat{\nu}) \subset K(\widehat{\mu})$ and hence $h_{\widehat{\nu}}(\phi) \leq h_{\widehat{\mu}}(\phi)$ ($0 \leq \phi \leq 2\pi$). Let $F := \widehat{\nu}/\widehat{\mu}$. Since $\widehat{\mu}$ and $\widehat{\nu}$ are of completely regular growth, there is a set $E \subset (0, \infty)$ of linear density zero such that

$$\lim_{\substack{r \rightarrow \infty \\ r \notin E}} \frac{\log |F(re^{i\phi})|}{r} = h_{\widehat{\nu}}(\phi) - h_{\widehat{\mu}}(\phi) \leq 0$$

uniformly in ϕ . Hence

$$(2) \quad \lim_{\substack{r \rightarrow \infty \\ r \notin E}} \frac{\log M(r, F)}{r} = 0.$$

It is now easy to show that (2) holds without the restriction $r \notin E$ because $M(r, F)$ is an increasing function of r , and since $(r, 2r) \setminus E$ is nonempty for all sufficiently large r . This proves the lemma.

We also need the following lemma (see, e.g., [8, p. 51]).

LEMMA 2. *Let f be an entire function of exponential type zero. If f is bounded on some line then f is constant.*

2.2. Proof of the theorem. Fix $\vartheta_0 \in \Theta$ and $t \in \mathbf{R}^k$. For $\zeta \in \mathbf{C}$, let

$$\varphi(\zeta) = \widehat{P}_{\vartheta_0}(\zeta t) = \int e^{i\zeta \langle t, x \rangle} dP_{\vartheta_0}(x), \quad \psi(\zeta) = \widehat{gP}_{\vartheta_0}(\zeta t) = \int e^{i\zeta \langle t, x \rangle} g(x) dP_{\vartheta_0}(x),$$

where $\langle t, x \rangle$ denotes the usual inner product of $t, x \in \mathbf{R}^k$.

We want to show that

$$(3) \quad \psi(\zeta) = \gamma(\vartheta_0) \varphi(\zeta) \quad (\zeta \in \mathbf{C}).$$

We first show that the meromorphic function ψ/φ has no poles. To this end, let ζ be a p -fold zero of φ , i.e.,

$$0 = \varphi^{(l)}(\zeta) = \int \left(i \langle t, x \rangle \right)^l e^{i\zeta \langle t, x \rangle} dP_{\vartheta_0}(x) \quad (l = 0, \dots, p-1).$$

It follows that for every $\vartheta \in \mathbf{R}^k$,

$$\begin{aligned} \int \left(i \langle t, x \rangle \right)^l e^{i\zeta \langle t, x \rangle} dP_{\vartheta}(x) &= \int \left(i \langle t, x + \vartheta - \vartheta_0 \rangle \right)^l e^{i\zeta \langle t, x + \vartheta - \vartheta_0 \rangle} dP_{\vartheta_0}(x) \\ &= e^{i\zeta \langle t, \vartheta - \vartheta_0 \rangle} \sum_{j=0}^l \binom{l}{j} \left(i \langle t, \vartheta - \vartheta_0 \rangle \right)^{l-j} \varphi^{(j)}(\zeta) = 0 \\ & \quad (l = 0, \dots, p-1). \end{aligned}$$

In particular, $x \rightarrow (i(t, x))^l e^{i\zeta(t, x)}$ is an unbiased estimator of zero for the model \mathcal{P} for $l = 0, \dots, p - 1$. Since g is a UMVU estimator, this implies (see, e.g., [6, p. 77])

$$0 = \int \left(i(t, x) \right)^l e^{i\zeta(t, x)} g(x) dP_{\vartheta_0}(x) = \psi^{(l)}(\zeta) \quad (l = 0, \dots, p - 1).$$

Thus ζ is a zero of ψ with multiplicity at least p . Hence

$$(4) \quad \frac{\psi}{\varphi} \text{ is an entire function.}$$

It is easy to see that $\varphi = \widehat{\mu}$ and $\psi = \widehat{\nu}$, where μ is a probability measure and ν is a signed measure on the Borel sets of \mathbf{R} , both having compact support and satisfying

$$(5) \quad \text{supp } \nu \subset \text{supp } \mu.$$

We conclude from (4), (5), and Lemma 1 that ψ/φ is of exponential type zero.

Since g is bounded on the support of P_{ϑ_0} , we have for every $\eta \in \mathbf{R}$

$$\left| \psi(i\eta) \right| \leq \int e^{-\eta(t, x)} \left| g(x) \right| dP_{\vartheta_0}(x) \leq \sup \left\{ \left| g(x) \right| : x \in \text{supp } P_{\vartheta_0} \right\} \varphi(i\eta).$$

Hence ψ/φ is bounded on the imaginary axis. By Lemma 2, ψ/φ is constant. Since $\psi(0)/\varphi(0) = \gamma(\vartheta_0)$, (3) is proved.

Setting $\zeta = 1$ in (3), we see that the Fourier transforms of the signed measures gP_{ϑ_0} and $\gamma(\vartheta_0)P_{\vartheta_0}$ are equal at t . Since $t \in \mathbf{R}^k$ was arbitrary, $gP_{\vartheta_0} = \gamma(\vartheta_0)P_{\vartheta_0}$ and thus $g(x) = \gamma(\vartheta_0)P_{\vartheta_0}$ -almost surely.

2.3. Proof of the corollary (sketch). First conclude that g is locally integrable with respect to Lebesgue measure. Take any compactly supported continuous function $\alpha: \mathbf{R}^k \rightarrow \mathbf{R}$ and consider the convolution $g_\alpha := \alpha * g$. Check that g_α is a continuous UMVU estimator for $\alpha * \gamma$. The theorem together with Remark 2 shows that g_α is a constant function. The assertion follows by letting α tend to the Dirac measure.

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SPECTRAL PROPERTIES OF SAMPLE COVARIANCE MATRICES*

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(Translated by the author)

Abstract. The expectation value of the resolvents of sample covariance matrices and the variance of their matrix elements are investigated. It is assumed only that variables have zero expectation values and the maximal fourth moment of variables exists. The principal spectral equations obtained earlier only in the form of limit formulas are derived with upper estimates of the remainder terms, accurate up to absolute constants. The remainder terms prove to be small for large sample size and a small value of a special measure of the quadratic forms variance.

Key words. spectral functions, sample covariance matrix, multivariate analysis, increasing dimension

1. Introduction. Let $\mathbf{x} \in \mathbf{R}^n$ be an observation vector, $\mathcal{X} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$ be a sample of size N , $\bar{\mathbf{x}}$ be the sample average vector, and

$$\mathbf{S} = N^{-1} \sum_{m=1}^N \mathbf{x}_m \mathbf{x}_m^T, \quad \mathbf{C} = N^{-1} \sum_{m=1}^N (\mathbf{x}_m - \bar{\mathbf{x}}) (\mathbf{x}_m - \bar{\mathbf{x}})^T$$

be sample covariance matrices constructed by the known expectation vector $\boldsymbol{\mu} = \mathbf{E}\mathbf{x} = \mathbf{0}$ (matrices \mathbf{S}) and by the unknown $\boldsymbol{\mu}$ (matrices \mathbf{C}). In 1967, Marchenko and Pastur [1] investigated spectral properties of the resolvent of operators that can be represented by the Gram matrices \mathbf{S} of increasing dimensions $n \rightarrow \infty$ when $N = N(n) \rightarrow \infty$ and $n/N \rightarrow \lambda > 0$. They established the convergence of the normed traces of these resolvents and obtained the limit expressions. In 1968–1970, Kolmogorov and Blagoveshchenskii suggested using such asymptotics for studying the spectral properties of the sample covariance matrices and solving applied problems [2], [3]. In [2], for $\mathbf{x} \sim \mathcal{N}(0, \mathbf{I})$, where (and below) \mathbf{I} is identity matrix, the existence of limits $M_k = p \lim_{n \rightarrow \infty} \text{tr } \mathbf{C}^k$ was established and the recurrent relations for M_k were found. In [3], for two normal populations with common covariance matrix, the asymptotic expansion was obtained and a limit formula for probability of classification errors using Wald's sample linear discriminant function was derived. In the monographs by Girko, [4] (1975) and [5] (1988), the limit theory of spectral properties of increasing dimensions random matrices from wide classes was developed.

For matrices of the form \mathbf{S} in [4] and [5], a remarkable feature was established: assuming components of the vector \mathbf{x} to be independent, under some weak restrictions of the Lindeberg condition type, the matrix elements of the resolvent of \mathbf{S} converge in probability to functions that depend only on the limit value of the ratio n/N and the limit spectral functions of matrices $\boldsymbol{\Sigma} = \mathbf{E}\mathbf{S} = \text{cov}(\mathbf{x}, \mathbf{x})$ independently of the other distribution characteristics. In [6] and [7] (1983), for normal distributions under the same asymptotics, a "principal spectral equation" (of the form of (4) and (5); see below) was found, relating functionally the limit spectral functions of the matrices $\boldsymbol{\Sigma}$, \mathbf{S} , and \mathbf{C} . (In the normal case, the matrices \mathbf{C} may be

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